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Learning temporal nodes Bayesian networks

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ABSTRACT

Temporal nodes Bayesian networks (TNBNs) are an alternative to dynamic Bayesian networks for temporal reasoning with much simpler and efficient models in some domains. TNBNs are composed of temporal nodes, temporal intervals, and probabilistic dependencies. However, methods for learning this type of models from data have not yet been developed. In this paper, we propose a learning algorithm to obtain the structure and temporal intervals for TNBNs from data. The method consists of three phases: (i) obtain an initial approximation of the intervals, (ii) obtain a structure using a standard algorithm and (iii) refine the intervals for each temporal node based on a clustering algorithm. We evaluated the method with synthetic data from three different TNBNs of different sizes. Our method obtains the best score using a combined measure of interval quality and prediction accuracy, and a competitive structural quality with lower running times, compared to other related algorithms. We also present a real world application of the algorithm with data obtained from a combined cycle power plant in order to diagnose temporal faults.

1. Introduction

Bayesian Networks [18] have been studied as a popular technique to deal with uncertainty. They have proven to be successful in different applications, such as medical diagnosis, crime risk factor analysis, sensor validation and customer analysis [19,20,12,7], among others. However, traditional Bayesian Networks cannot deal with temporal information. For this reason, an extension called Dynamic Bayesian Networks (DBNs) [5] was introduced. DBNs can be seen as multiple slices of a *static* BN over time, with links between adjacent slices. Nonetheless, these models may become quite complex, in particular when only a few important events occur over time.

Temporal Nodes Bayesian Networks (TNBNs) [1] are another extension of Bayesian Networks. They belong to a class of temporal models known as *Event Bayesian Networks* [10]. TNBNs were proposed to manage uncertainty and temporal reasoning. In a TNBN, each Temporal Node has intervals associated to it. Each node represents an event or state change of a variable. An arc between two Temporal Nodes corresponds to a causal-temporal relation. One interesting property of this class of models, in contrast to Dynamic Bayesian Networks, is that temporal intervals can differ in number and size.

TNBNs have been used in diagnosis and prediction of temporal faults in a steam generator of a fossil power plant [1]. The problem is that there does not exist an algorithm to learn TNBNs models, so the model has to be obtained from external sources such as domain experts. This can be a hard and prone to error task.

In this paper, we propose a learning algorithm to obtain the structure and the temporal intervals for TNBNs from data. The learning algorithm consists of three phases. In the first phase, we obtain an approximation of the intervals using a simple algorithm such as equal-width discretization (EWD) or a K-means clustering approximation. For the second phase, the BN structure is obtained with the structure learning algorithm introduced in [3]. The last step is performed to refine the intervals for each Temporal Node. This refinement uses the structure obtained in the previous step in order to improve the

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intervals. For this, our algorithm obtains a number of possible sets of intervals for each configuration of the parents. We used a clustering approach to obtain the intervals, more specifically the algorithm models the data based on a Gaussian mixture model. Each component of the mixture corresponds to an interval. Given that our algorithm obtains different intervals, we applied two pruning techniques to discriminate some intervals and to increase the efficiency of our algorithm. From the remaining intervals, our algorithm selects the set of intervals that maximizes the prediction accuracy.

We evaluate our algorithm with three synthetic examples of TNBNs. The data was generated with different distributions. We compare our algorithm with two baselines: K-means and Equal-width discretization. We also compare it to the algorithm proposed in [9]. In the experiments, our algorithm obtains the best score in terms of a combined measure that takes into account predictive accuracy and interval quality. Our algorithm also obtained lower running times. Finally, we present an application of the algorithm with real data obtained from a power plant simulator in order to diagnose and predict temporal faults.

The paper is organized as follows. In Section 2, the Temporal Nodes Bayesian Network model is described. In Section 3, we present related work about learning TNBNs. In Section 4, we introduce the proposed learning algorithm called LIPS (Learning Intervals-Parameters-Structure). In Section 5, we present an overview of experiments and measures used to evaluate our algorithm. The results of the algorithm with synthetic data are described in Section 6. In Section 7, we show the results of applying LIPS to a real world application. In Section 8, we present our conclusions and ideas for future research.

2. Temporal nodes Bayesian networks

Bayesian networks (BNs) are a successful model for dealing with uncertainty. However, *static* BNs are not suited to deal with temporal information. For this reason, Dynamic Bayesian Networks [5,4] were introduced. In a DBN, a copy of a base model is made for each time stage. These copies are linked via a transition network. In this transition network it is common that only links between consecutive stages are allowed (Markov property). The problem is that DBNs can become very complex. This is unnecessary when dealing with problems for which there are only a few changes for each variable in the model. Moreover, DBNs are not capable of managing different levels of time granularity. They usually have a fixed time interval between stages.

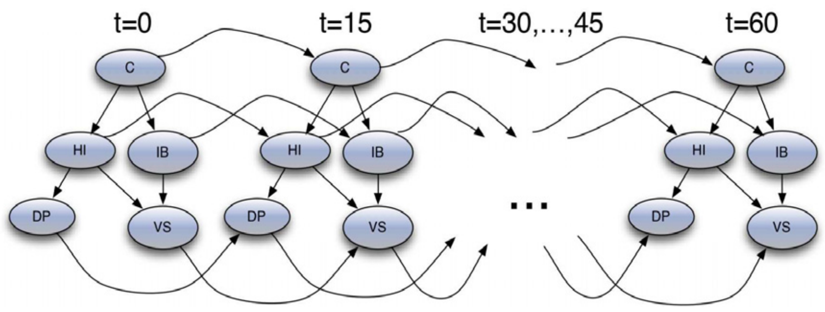
A Temporal Nodes Bayesian Network (TNBN) [1,10] is composed of a set of Temporal Nodes (TNs). TNs are connected by edges, each edge represents a causal-temporal relationship between TNs. There is at most one state change for each variable (TN) in the temporal range of interest. The value taken by the variable represents the interval in which the event occurs. Time is discretized in a finite number of intervals, allowing a different number and duration of intervals for each node (multiple granularity). Each interval defined for a child node represents the possible delays between the occurrence of one of its parent events (cause) and the corresponding child event (effect). Some Temporal Nodes do not have temporal intervals, these correspond to Instantaneous Nodes. Root nodes are instantaneous by definition [1]. Formally, a TNBN is defined as follows.

Definition 1. A TNBN is defined as a pair $B = (G, \Theta)$. G is a Directed Acyclic Graph, $G = (\mathbf{V}, \mathbf{E})$. G is composed of $\mathbf{V}$, a set of Temporal and Instantaneous Nodes; $\mathbf{E}$ a set of edges between Nodes. The Θ component corresponds to the set of parameters that quantify the network. Θ contains the values $\Theta_{v_i} = P(v_i|Pa(v_i))$ for each $v_i \in \mathbf{V}$; where $Pa(v_i)$ represents the set of parents of v_i in G.

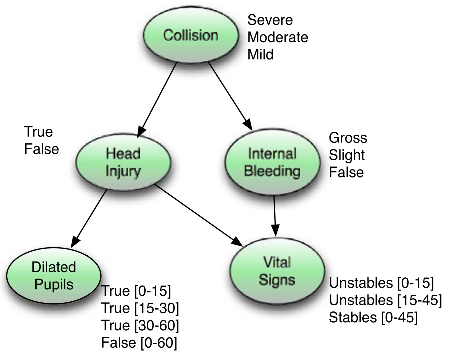
Definition 2. A Temporal Node, v_i, is defined by a set of states $\mathbf{S}$, each state is defined by an ordered pair $S = (\lambda, \tau)$, where λ is the value of a random variable and $\tau = [a, b]$ is the interval associated, with an initial value a and a final value b.These values correspond to the time interval in which the state change occurs. In addition, each Temporal Node contains an extra default state $s =$ (“no change”, $\emptyset$), which has no interval associated. If a Node has no intervals defined for any of its states, then it receives the name of Instantaneous Node.

The following is an example based on [1], its corresponding graphical representation as a DBN and a TNBN are shown in Fig. 1.

Example 1. Assume that at time $t = 0$, an automobile accident occurs, a Collision. This kind of accident can be classified as *severe*, *moderate*, or *mild*. To simplify the model, we will consider only two immediate consequences for the person involved in the collision, Head Injury and Internal Bleeding. Head Injury can bruise the brain, and chest injuries can lead to internal bleeding. These three are instantaneous events that will generate subsequent changes, for example the Head Injury event might generate Dilated Pupils and unstable Vital Signs. Suppose that we gathered information about accidents occurred in a specific city. The information indicates that there is a strong casual relationship between the severity of the accident and the immediate effect of the patient’s state. Additionally, a physician domain expert provided some important temporal information: If a head injury occurs, the brain will start to swell and if left unchecked the swelling will cause the pupils to dilate within 0 to 60 min. If internal bleeding begins, the blood volume will start to fall, which will tend to destabilize vital signs. The time required to destabilize vital signs will depend on the bleeding severity: if the bleeding is gross, it will take



(a) A DBN



(b) A TNBN

Fig. 1. A comparison between (a) DBN and (b) TNBN for a simple example. In this example, C=Collision, HI= Head Injury, IB= Internal Bleeding, DP= Dilated Pupils, and VS=Vital Signs. In the DBN there are copies over time of the static network. In contrast, the structure of the TNBN is simpler and contains intervals in the nodes in order to manage temporal information. For instance, it says that the pupils can be dilated in three possible time intervals depending on the severity of the accident.

from 0 to 15 min; if the bleeding is slight it will take from 15 to 45 min. A head injury also tends to destabilize vital signs, taking from 0 to 15 min to make them unstable.

In Fig. 1(a) a DBN representation is presented for this example, the DBN is presented as a series of static BNs connected with edges between them. For this example, we used a slice of 15 minutes, the maximum common divisor of the time intervals. In this case, the unrolled DBN contains five static BNs with a total of 25 nodes. This is a simple DBN which considers the following assumptions: (i) a state only depends on the previous one, (ii) there are only links between the same variable at different time slices.

In contrast, in Fig. 1(b) a TNBN for the same problem is presented. The model presents three instantaneous nodes: Collision, Head Injury and Internal Bleeding. These events will generate subsequent changes that are not immediate: Dilated Pupils and unstable Vital Signs, that depend on the severity of the accident therefore they have temporal intervals associated to them. This TNBN contains only five nodes, in contrast to the 25 nodes of the DBN.

In TNBNs, each variable represents an event or state change. So, only one (or a few) instance(s) of each variable is required, assuming there is one (or a few) change(s) of a variable state in the temporal range of interest. No copies of the model are needed, and no assumption about the Markovian nature of the process is made. TNBNs can deal with multiple granularity, because the number and the size of the intervals for each node can be different.

3. Related work

There are several methods to learn BNs from data [17]. Unfortunately, the algorithms used to learn BNs cannot deal with the problem of learning temporal intervals. Then, these cannot be applied directly to learn TNBNs.

To the best of our knowledge, there is only one previous work that attempts to learn a TNBN. [15] proposed a method to build a TNBN from a *temporal probabilistic database*. The method obtains the network structure from a set of temporal dependencies in a *probabilistic* temporal relational model (PTRM). A Temporal Relational Model (TRM) is a relational model with temporal attributes. A PRTM is a TRM extended by adding a probabilistic attribute *p*. In order to build the TNBN, they obtain a variable ordering that maximizes the set of conditional independence relations implied by a dependency graph obtained from the PTRM. Based on this order, a directed acyclic graph corresponding to the implied independence relations is obtained. This graph represents the structure of the TNBN. They assume a known probabilistic temporal relational model from the domain of interest, which is not always known. Building this PTRM can be as difficult as building a TNBN, and they do not learn the temporal intervals of each node. In contrast, our approach constructs the TNBN directly from data and it also learns the intervals for each node.

Algorithm 1: Algorithm introduced in [9]

```
   Data: An initial discretization.
   Result: A local optimal discretization.
1  Push all continuous variables onto a queue $Q$
2  while $Q$ is not empty do
3      Remove first element $X$ from $Q$
4      Compute new discretization policy for $X$
5      if score with new discretization $<$ score with old discretization then
6          Use new discretization
7          For all $Y$ interacting with $X$, if $Y \notin Q$, push $Y$ onto $Q$.
8      end
9  end
10 return Discretization policy
```

Another related work is described in [9]. Here the idea is to learn the structure of a BN while discretizing the continuous variables. For this, a score based on the Minimum Description Length [14] is proposed. The score takes into account the parameters, the structure, and the discretization policy. With this score, a search for the best discretization is done for each continuous variable. The algorithm scores the N_i midpoints of each variable. Then, a top-down refinement is performed using a greedy search strategy. Depending on the network structure, this approach can become complex if the continuous variables interact with each other (if a variable does not interact with other variables it can be discretized separately). For this reason, the approach consists on discretizing one variable at a time and leave the rest as discrete and fixed. Therefore, the algorithm does guarantee finding an optimal discretization. In order to learn the structure of the network, all the nodes are first discretized and later, the structure learning algorithm is applied. This alternating process is performed until convergence.

The algorithm presented in [9] for discretizing several variables is shown in Algorithm 1, and it proceeds as follows. First, it pushes all continuous variables onto a queue Q, then a cycle starts while Q is not empty. A discretization is obtained for the considered variable in a greedy way. Then, the interacting variables are pushed onto Q. The algorithm finishes when the queue is empty. This algorithm was proposed to learn the structure of a BN while discretizing continuous variables. This problem is similar to the one of learning the structure and the intervals for the TNBN thus we use it as a comparison for our experiments: the continuous variables will represent the temporal nodes of the TNBN, the result will be a BN with some discretized nodes that will correspond to the intervals. Thus, the algorithm can be applied as usual. A problem with this algorithm is that it may become too complex in cases where the continuous data is sparse since it has to obtain all the midpoints in the data. This may also happen if the network has many edges (several nodes interacting with each other). In contrast, the LIPS algorithm is not directly affected by the data distribution.

4. Learning algorithm

In this section we first present the interval learning algorithm that is applied for each Temporal Node. We initially assume that we have a defined structure. Later, we present the whole learning algorithm. The proposed algorithm obtains a structure and sets of intervals for the TNBN. The algorithm assumes that the root nodes are instantaneous nodes and it obtains a finite number of non overlapping intervals for each temporal node. Our approach uses the times (delays) between parent events and the current node as input for learning the intervals. With this top-down approach the algorithm is guaranteed to arrive at a local maximum in terms of predictive score.

4.1. Interval learning

Initially, we assume that the events that we can model as TNs follow a known distribution. With this idea, we can use a clustering algorithm to work with temporal data. Each cluster corresponds, in principle, to a temporal interval. The first approximation of the algorithm ignores the values of the parent nodes. Later, we refine the method by incorporating the parent nodes configurations.

4.2. First approach: independent variables

Our approach uses a Gaussian mixture model (GMM) to perform an approximation of the data, therefore we can use the Expectation-Maximization (EM) algorithm [6] to obtain the parameters of the GMM. EM works iteratively using two steps: (i) The E-step tries to *guess* the parameters of the Gaussian distributions, (ii) the M-step updates the parameters of the model based on the previous step. By applying EM, we obtain a number of Gaussians (clusters), specified by their mean

Table 1
Initial sets of intervals obtained for node *Dilated Pupils*. There are three sets of intervals for each partition.

Partition	Intervals
Head Injury=true	[11−35]
	[11−27][32−53]
	[8−21][25−32][45−59]
Head Injury=false	[3−48]
	[0−19][39−62]
	[0−14][28−40][47−65]

and variance. For now, assume that the number of temporal intervals (Gaussians) is given. For each TN we have a dataset of points over time, and these are clustered using GMM, to obtain k Gaussian distributions. Based on the parameters of each Gaussian, the temporal intervals are initially defined as: $[\mu_k - \sigma_k, \mu_k + \sigma_k]$.

Now we deal with the problem of finding the number of intervals. The ideal solution has to fulfill two conditions: (i) the number of intervals must be small in order to reduce the complexity of the network, and (ii) the intervals should yield good estimations when performing inference over the network. Based on the above, our approach uses the EM algorithm varying the parameter for the number of clusters from 1 to ℓ, where ℓ is the highest value (for the experiments in this paper we used $\ell = 3$).

In order to select the best set of intervals, we evaluate the network. This corresponds to an indirect measure of the quality of the intervals. In particular, we used the *Brier Score* [2] to measure the predictive accuracy of the network. The selected set of intervals for each TN are those that maximize the Brier Score. The Brier Score is defined as

$$BS = \frac{1}{n}\sum_{i=1}^{n}(1 - P_i)^2$$

where P_i is the marginal posterior probability of the correct value of each node given the evidence. The BS is obtained by instantiating a random subset of variables in the model, predicting the unseen variables, and obtaining the BS for these predictions.

4.2.1. *An example*

We will illustrate the process of using the first approximation for obtaining the intervals for the TN *Dilated Pupils* of Fig. 5(a). We can see that its parent node (Head Injury) has two configurations, *true* and *false*. Thus, we separate the temporal data of Dilated Pupils in two partitions, one for each configuration of the parent node. Then, for each partition we apply the first approximation of the algorithm. We apply the EM algorithm to get a Gaussian mixture with parameters 1, 2 and 3 as the number of clusters. That give us six different sets of intervals, as we show in Table 1. Then we obtain the Brier Score for each set of intervals to measure its quality and select the set of intervals with the best score.

4.3. *Second approach: considering the network topology*

Now we will construct a more accurate approximation. For each Temporal Node we use the configurations of the parent nodes. The number of configurations of each node X is $q_X = \prod_{Y \in Pa(X)} |s_Y|$ (the product of the number of states of the parents nodes).

Formally, we construct partitions of the data (temporal data), one partition for each configuration. Then, we get binary combinations taking partitions p_i,p_j (see Fig. 2(a)). This will yield $q(q-1)/2$ different binary combinations. For each p_i, p_j the first approximation of the algorithm is applied. Therefore, we obtain ℓ intervals for each partition (see Fig. 2(b)). Then, a combination of the sets of the first approximation is performed (see Fig. 2(c)), this will generate ℓ^2 sets of intervals for each pair p_i, p_j. Lastly, these intervals need to be adjusted (see Fig. 2(d)) in order to be defined in an exhaustive and mutually exclusive set of intervals, this adjustment is performed by Algorithm 2.

Algorithm 2 performs a cycle, for each set of intervals we sort them by its starting point. Then, we check if there is an interval contained in another interval. While this is true, the algorithm obtains an average interval, taking the average of the start and end points of the intervals and replacing these two intervals with the new one. Next, we refine the intervals to be continuous by taking the mean of two adjacent values and finally a rounding operation is performed. A graphical representation of the interval learning algorithm is presented in Fig. 3. An example of this algorithm is presented in Section 4.3.2.

As in the first approximation, the best set of intervals for each TN is selected based on the predictive accuracy in terms of the BS. However, when a TN has as parents other Temporal Nodes (an example of this situation is illustrated in Fig. 5(b)), the state of the parents is not initially known. For this reason, we cannot directly apply the second approximation. In order to solve this problem, the intervals are sequentially selected in a top-down fashion according to the TNBN structure. That is, we first select the intervals for the nodes in the second level of the network. Once these are defined, we know the values of the parents of the nodes in the third level. Then, we can find their intervals; and so on, until the leaf nodes are reached.

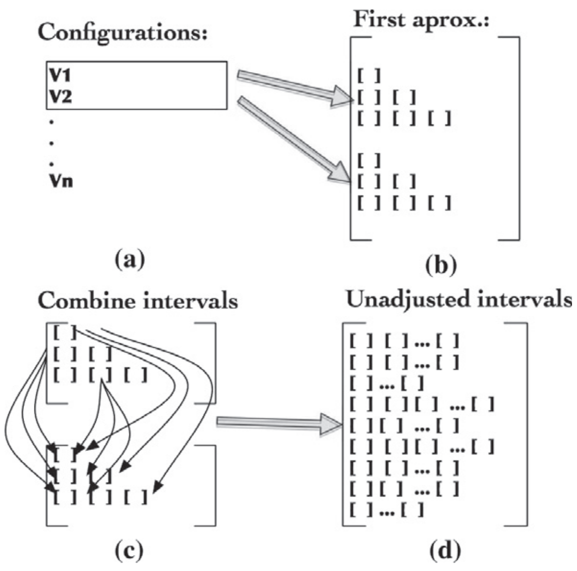


Fig. 2. Graphical description of the interval learning algorithm: (a) binary partitions that correspond to the configurations of the parent nodes are selected, (b) for each partition the first approximation to generate different sets of intervals is applied (between 1 and 3 in this paper), (c) the intervals generated in the previous step are combined, (d) these intervals are adjusted using Algorithm 2.

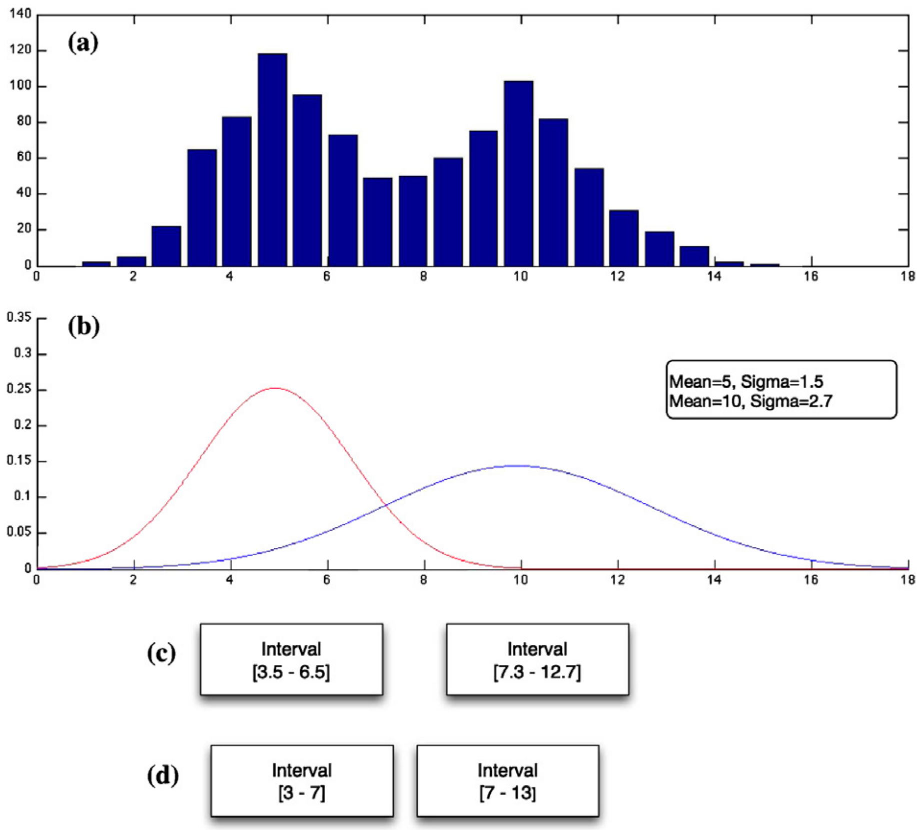


Fig. 3. Graphical representation of the interval learning algorithm: (a) histogram of the temporal data, (b) Gaussians obtained by applying the EM algorithm, (c) first approximation of the intervals using the parameters of the Gaussians, (d) final intervals of the algorithm.

Algorithm 2: Algorithm to adjust the intervals

```
    Data: Array of intervals sets
    Result: Array of adjusted intervals
 1  for Each set of intervals s do
 2      sortIntervalsByStart(s)
 3      while Interval i is contained in Interval j do
 4          tmp=AverageInterval(i,j)
 5          s.replaceInterval(i,j,tmp)
 6      end
 7      for $k = 0$ to number of intervals in set s -1 do
 8          Interval[k].end=round ((Interval[k].end + Interval[k+1].start)/2)
 9      end
10  end
```

Table 2
Final sets of intervals obtained for node *Dilated Pupils*. The selected set of intervals that obtained the best Brier Score is presented in bold.

Intervals node dilated pupils
[0—15][16—37][38—62]
[0—12][13—31][32—43][44—65]
[7—37][38—53]
[15—39][40—59]
[0—13][14—23][24—38][39—60]

4.3.1. Pruning

Taking the combinations and joining the intervals can become computationally expensive. The number of sets of intervals for a node is in $O(q^2\ell^2)$, where q is the number of configurations and ℓ is the maximum number of clusters for the GMM. For this reason we used two pruning techniques for each TN to reduce the computation time.

The first pruning technique discriminates the partitions that contain few instances. For this, we count the number of instances in each partition, and if it is greater than a value

$$\beta = \frac{\text{Number of instances}}{\text{Number of partitions} \times 2}$$

the configuration is used, if not it is discarded.

A second pruning technique is applied when the intervals for each combination are being obtained. If the final set of intervals contains only one interval (no temporal information) or more than α (producing a complex network), the set of intervals is discarded. In section 6.1 we present some experiments to obtain the value of α used in the rest of the paper.

4.3.2. Revisited example

We had six different sets of intervals obtained by the first approximation for the TN Dilated Pupils, as we show in Table 1. Then, we combine these sets of intervals for each partition with the other partitions. In this case, there could be $3 \times 3 = 9$ different sets of intervals, but some of these combinations are eliminated using the proposed pruning methods.

For example, we should merge the sets [11—35] and [3—48] because they correspond to different partitions, if we join them we get [11—35][3—48]. With this set we apply Algorithm 2. In this case when we sort them to get [3—48][11—35]. The next thing to do is to check whether one interval is contained in another, this is true ([11—35] in [3—48]), then we obtain the average interval [7—41], this would be the final interval, however because of the pruning methods, this interval will not be added to the final list of intervals, since it is composed of only one temporal interval. The algorithm continues taking [11—35] and [0—19][39—62]. If we join these intervals we get [11—35][0—19][39—62]. Applying Algorithm 2 we sort the intervals to obtain [0—19][11—35][39—62]. Here, no interval is contained in another so we skip the code inside the *while* statement. Then we refine the intervals to obtain [0—15][16—37][38—62], this set of intervals is added to the final list of intervals. The process continues with the rest of the nodes and we obtain the sets of intervals shown in Table 2.

This process is applied to each Temporal Node. To select the set of intervals for each node we apply the inference test (based on the maximization of the Brier Score) described in Section 4.2. In this case, the best set of intervals is: [0—15][16—37][38—62].

Applying the process to the remaining TN Vital Signs would yield the sets of intervals shown in Table 3. We present the best set of intervals in the TNBN of Fig. 4.

Table 3
Final sets of intervals obtained for node *Vital Signs*. The selected set of intervals that obtained the best Brier Score is presented in bold.

Intervals node vital signs
[**0**–**13**][**13**–**40**][**40**–**50**]
[0–6][7–29][30–43][44–50]

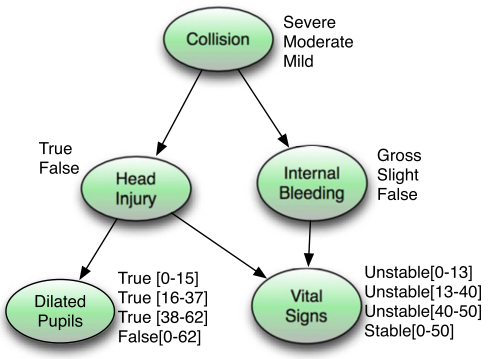


Fig. 4. A learned TNBN in the Example presented in Section 4.3.2.

4.4. Structure learning

Now we present the whole algorithm that learns the structure and the intervals of the TNBN. First, we perform an initial discretization of the temporal variables, for example using an Equal-width discretization or K-means [8] clustering approximation.

4.4.1. Equal-width discretization

The simplest approximation for obtaining intervals is to apply an equal-width discretization in all the temporal nodes. For this, we set n as the number of intervals. Then a discretization is performed to obtain $n - 1$ division points in the sorted data. This yields to n intervals of the same size.

4.4.2. K-means approximation

Since equal-width discretization is a very simple approximation, we proposed a second method, based on K-means clustering. For this, we apply the K-means algorithm with n number of clusters. The result of K-means consists of n points that correspond to the centroids of the clusters. Then, we convert these centroids into intervals using Algorithm 3.

Algorithm 3: Algorithm that converts *K-means* centroids into intervals

```
   Data: Pi centroid points obtained by k-means, continuous values data from a temporal node.
   Result: Set of intervals for a temporal node
 1 min=minimum(data)
 2 max=maximum(data)
 3 Interval[0].start=min
 4 Interval[0].end=average(Pi[0],Pi[1])
 5 for i=0 to size(Pi)-2 do
 6     Interval[i+1].start=average(Pi[i],Pi[i+1])
 7     Interval[i+1].end=average(Pi[i+1],Pi[i+2])
 8 end
 9 i=size(Pi)-1
10 Interval[i].start=average(Pi[i],Pi[i+1])
11 Interval[i].end=max
```

Algorithm 3 has as parameters the result of the K-means clustering process (centroids), and the corresponding data values. The algorithm obtains the first interval taking the minimum value of the data and the average of the first two centroids. Then, a cycle is performed to obtain the rest of the intervals except the last one. These intervals are obtained by taking the average of the centroids. The last interval is obtained by taking the average of the last two centroids and the maximum value of the data.

Using either equal-width discretization or K-means clustering approximation we obtain an initial approximation of the intervals for all the Temporal Nodes, and we can perform a standard BN structural learning. We decided to use the K2

Table 4
Collected data to learn the TNBN of Fig. 5(a). For Dilated Pupils and Vital Signs the temporal data represents the minutes after the collision occurred.

Collision	Head injury	Internal bleeding	Dilated pupils	Vital signs
Severe	True	Gross	14	20
Moderate	True	Gross	25	25
Mild	False	False	–	–
…	…	…	…	…
Severe	True	Gross	[10–20]	[15–30]
Moderate	True	Gross	[20–30]	[15–30]
Mild	False	False	–	–
…	…	…	…	…

algorithm [3], since K2 has as a parameter an ordering of the nodes, this ordering of the nodes affects the structure of the network since it restricts the possible arcs in the following way. If a node x_i precedes x_j in the ordering, then structures in which there is an arc from x_j to x_i are not allowed. For learning TNBNs we can exploit this parameter and define an order based on domain information. For instance, for the medical example presented in Section 2, we have a partial ordering of the temporal events: {Collision}, {Head Injury, Internal Bleeding}, {Edema, Blood Pressure, Heart Rate}, and {Dilated Pupils, Shock}. Therefore, we can define a total ordering based on the partial one, this ordering is then used in the K2 algorithm.

Once the network structure has been obtained, we apply the interval learning algorithm described in Section 4.1. Moreover, this process of alternating interval learning and structure learning may be iterated. For the experiments presented in this paper we only applied one iteration of the algorithm.

4.4.3. A complete example

First assume we have data from accidents that is similar to the one presented in the upper part of Table 4. The first three columns have nominal data, however, the last two columns have temporal data that represents the occurrence of those events after the collision. Those two columns would correspond to Temporal Nodes of the TNBN. We start by applying the EWD algorithm in the numerical data, so it would yield results as the ones presented in lower part of Table 4.

Using the discretized data we can apply the K2 learning algorithm using the partial ordering of the temporal events: {Collision}, {Head Injury, Internal Bleeding}, and {Dilated Pupils, Vital Signs}. Now we have an Initial TNBN, however the obtained intervals are somewhat naive, so interval learning algorithm can be applied as presented in the examples in sections 4.2.1 and 4.3.2. This process will learn a TNBN like the one presented in Fig. 4.

5. Evaluation on synthetic cases

In order to evaluate our algorithm, we considered three synthetic cases corresponding to the TNBNs presented in Fig. 5. In Fig. 5(a), a simple model of the consequences of a car accident is presented. This network contains two temporal nodes and five nodes in total. In Fig. 5(b) an expanded model of the car accident is presented, it contains five temporal nodes and eight nodes in total. The third TNBN is shown in Fig. 5(c) and it corresponds to a TNBN used to diagnose faults in a subsystem of a fossil power plant [10], it contains 10 temporal nodes and 14 nodes in total.

Given that we know the original structure, parameters, and intervals of the reference TNBNs, we can use it to compare the results of our algorithm. The idea is to sample the fully specified TNBN to generate data that reflects the model and the intervals. Then, we can use that data to reconstruct the model with the proposed algorithm. For the data that represents the intervals, we perform two experiments. In the first one, data was generated by a normal distribution over the intervals with parameters

$$\mu = \frac{Interval\_Start + Interval\_End}{2}$$

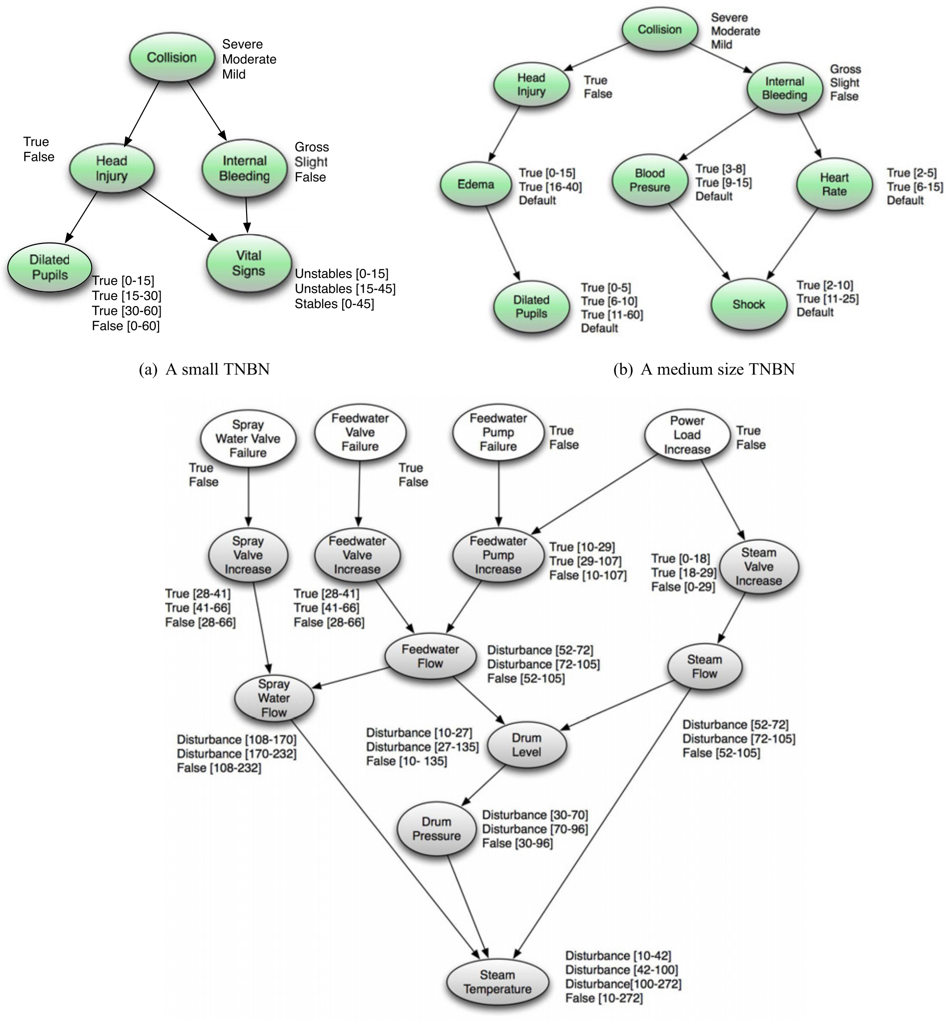
$$\sigma = \frac{Interval\_End - Interval\_Start}{2}$$

For the second test, the data was generated with a uniform distribution over the intervals. The intervals used to generate the data are the same as those shown in Fig. 5.

In the experiments, we compared the proposed algorithm (LIPS) and the algorithm introduced in [9] (F-G). As baselines we used an Equal-Width discretization (EWD) and a K-means approximation (K-M) (presented in Section 4.4) for learning the intervals for the Temporal Nodes.

5.1. Evaluation measures

Three different aspects of the TNBNs were evaluated: the structure (the graph), the intervals, and the complete model.



(a) A small TNBN (b) A medium size TNBN

(c) A large TNBN

Fig. 5. Three TNBNs of different size used in the experiments: (a) small, (b) medium, (c) large.

5.1.1. *Measures of structure quality*

Since we have a reference network, the most common approach for evaluating the learned network is to compare these networks. The learned network obtains a higher score as its degree of similarity with the reference network increases. For measuring the quality of the structure of the TNBN with respect to the reference network, three measures were used:

- The structural similarity (SS) [21], which is a score between 0 and 1 (maximum) that counts the similar edges with a reference network. Formally, let C be the adjacency matrix $n \times n$ of graph G, let $s(C) = \sum_{i,j} c_{i,j}$ the sum of all the components of matrix C. Even more, we define the conjunction $\bigwedge$ of two adjacency matrices C and C' as $D = C \bigwedge C'$, with $d_{i,j} = c_{i,j} \wedge c'_{i,j}$.The structural similarity for two graphs G and G' is defined as $SS(G, G') = \frac{s(C \wedge C')}{s(C)}$, where C is the adjacency matrix of G and C' is the adjacency matrix of G'.
- The number of added directed edges (E+) with respect to the reference network.
- The number of deleted directed edges (E−) with respect to the reference network.

The best network should obtain score 1 in structure similarity and 0 for edges deleted and added.

5.1.2. *Measures of interval quality*

The main difference between TNBNs and standard BNs are the temporal nodes and their intervals. For this reason it is important to evaluate their quality. For this, we used two measures.

- The relative temporal error (RTE). First we define the expected time as:

  $$t_e = (t_{end} + t_{ini})/2$$

  where t_{ini} and t_{end} are the initial and final values of the interval, so the expected time is the average of them. The range of a temporal node T is the difference between the maximum and the minimum value of all the intervals in the node. The relative temporal error for a Temporal Node T with respect to the original value t_{orig} is defined as:

  $$RTE = \frac{|t_e - t_{orig}|}{range(T)}$$

  this is the difference between the real event (original data) and the expected mean of the interval, normalized by the range of T.
- The number of intervals (# I) in the network.

The best model should obtain a value closer to 0 in temporal error because this would show that intervals are representative of the data. A lower number of intervals is also desired to reduce the complexity of the network.

5.1.3. Global measures

For evaluating the complete model our first measure is the Brier Score. The Brier score is defined as

$$BS = \frac{1}{N}\sum_{i=1}^{N}(1 - P(v_i|\mathbf{e}))^2$$

this is the sum of the quadratic error inferred for a node v_i with the given evidence $\mathbf{e}$.

For obtaining the BS for each learned network the following methodology was performed. For each instance, select a random subset of nodes of size s; $1 \leq |s| \leq N - 1$. Instantiate all the nodes selected with respect to the original data. Perform inference in the rest of the nodes (the ones not assigned). A perfect score for the BS will be a value of 0.

In earlier experiments [11] we used the measures mentioned before, however the results obtained showed that the relative time error and the Brier score were related in some way. We believe this happened due to the fact that some algorithms obtained a small number of large intervals thus, yielding large temporal error, but with high predictive score (because the probability of error was lower due to the larger intervals). Therefore, we propose another measure that combines relative time error with the predictive score. The measure called Predictive Temporal Error (PTE) is defined as:

$$PTE = \delta \times (RTE) + (1 - \delta) \times (BS)$$

where RTE is the relative time error, and BS is the Brier Score. Both of these measures are normalized between 0 and 1 and their best result is 0. The same happens with PTE. For our experiments we decided to use $\delta = 0.5$ to give the same weight to interval and prediction quality.

Even with this measure, the question of which is the best model remains open. We believe that the answer is related to the application domain. For some domains we could need models to be very effective in terms of accuracy, even if it is a complex model.

5.2. Measure of statistical significance

In order to evaluate if the results obtained by our algorithm were statistically significant the Kruskal–Wallis test [13] was performed. Kruskal–Wallis is a non-parametric test used to verify whether a group of data is generated from the same distribution. This test does not assume normality of the data and it is an extension of the Mann–Whitney–Wilcoxon test for three or more groups.

To apply this test the K samples are combined and arranged in increasing size order and given a rank number. When ties occur, the mean of the available rank numbers is used. The rank sum for each of the K samples is calculated. The test statistic is:

$$H = \left(\frac{12}{N(N+1)}\sum_{j=1}^{K}\frac{R_i^2}{n_j}\right) - 3(N+1) \tag{1}$$

where: n_j is the number of observations in group j, R_j is the rank sum of the j group. N is the size of the combined sample. The null hypothesis of equal means is rejected when H exceeds the critical value.

Using Kruskal–Wallis we know whether a pair of groups is significantly different or not, but we are not certain which of those pairs of groups are. For this we can perform a pair comparison using the Bonferroni adjustment. For a simple Bonferroni adjustment we divide the p value to be achieved for significance by the number of paired comparisons to be done. The number of total pairs is given by $\mathcal{T} = K(K-1)/2$, therefore for each pair comparison if the result is smaller than $\alpha_{kw}/\mathcal{T}$, (where α_{kw} is the significance level) then it is significant.

Table 5
Results varying α and the number of data points for the three reference TNBNs. BS is Brier Score, RTE is relative temporal error and SS is structural similarity.

α	Data								
	250			500			1000		
	BS	RTE	SS	BS	RTE	SS	BS	RTE	SS
Small TNBN									
2	0.24	0.32	0.60	0.23	**0.30**	0.63	0.23	**0.29**	**0.85**
3	0.24	**0.30**	**0.63**	0.23	**0.30**	0.60	0.23	0.31	**0.85**
4	**0.23**	0.33	**0.63**	0.23	**0.30**	**0.67**	0.23	**0.29**	**0.85**
5	0.24	**0.30**	**0.63**	0.23	0.32	0.63	0.23	0.30	0.83
6	0.25	0.34	0.60	0.23	0.31	0.63	**0.22**	0.31	0.83
	300			500			1000		
	BS	RTE	SS	BS	RTE	SS	BS	RTE	SS
Medium TNBN									
2	**0.21**	0.30	0.81	0.22	0.30	0.83	**0.21**	**0.28**	**0.96**
3	**0.21**	0.30	**0.88**	**0.20**	**0.29**	**0.94**	0.22	0.29	0.94
4	**0.21**	0.30	**0.88**	0.21	**0.29**	0.92	**0.21**	0.30	**0.96**
5	0.22	0.30	0.80	0.20	0.31	0.85	0.22	0.31	0.79
6	0.22	0.30	0.79	0.20	0.30	0.77	0.21	0.32	0.77
	500			750			1000		
	BS	RTE	SS	BS	RTE	SS	BS	RTE	SS
Large TNBN									
2	**0.13**	0.22	**0.18**	0.17	**0.22**	**0.25**	0.17	0.23	0.31
3	0.16	0.22	**0.18**	0.17	**0.22**	**0.25**	**0.16**	**0.21**	**0.35**
4	0.16	0.23	**0.18**	0.16	**0.22**	**0.25**	**0.16**	0.22	**0.35**
5	**0.13**	0.22	0.17	**0.13**	0.24	0.22	0.19	0.24	0.29
6	**0.13**	0.22	0.17	0.14	0.25	0.23	0.20	0.23	0.29

6. Experiments

The experiments were performed using synthetic data obtained from three different TNBNs of different sizes: small, medium and large.[1] These are depicted in Fig. 5. These TNBNs were used for the experiments since they are similar in terms of parameters to the ones presented in [1,10], two of them represent a simple medical problem and the last one was used in a real world application in a power plant.

6.1. Parameters

The LIPS algorithm makes use of different parameters such as α, the maximum number of intervals in the TNBN that is used during the pruning phase, ℓ the maximum number of initial intervals, and β is the minimum value of the number of instances needed in a partition to be considered through the algorithm.

For the ℓ parameter we decided to set $\ell = 3$ since this parameter has a direct effect on the complexity of the algorithm. The parameter $\beta = \frac{\text{Number of instances}}{\text{Number of partitions} \times 2}$, was explained in Section 4.3.1.

For the α parameter we performed different experiments with the three reference TNBNs. For each TNBN we varied the parameter from 2 to 6, we varied the number of data points considered and we evaluate the learned TNBNs with three measures: Brier Score, Relative Time Error and Structural Similarity. Each result, presents the average of 5 experiments using data with a Gaussian distribution and 5 experiments with a uniform distribution.

The results are summarized in Table 5, from which we can obtain the following conclusions. For the first TNBN the best results were obtained with $\alpha = 4$ since it obtained 6 of the best scores from the 9 measures. However, it is important to note that the results do not vary considerably for the other values of α. For the second net, the best results are obtained with $\alpha = 3$ and 4, both obtaining 5 best scores. One interesting aspect to note is that the structural similarity measure is the one that varies the most when α changes. For the third net the best results are obtained with $\alpha = 3$, and the second best parameter is with $\alpha = 4$. One important observation is that the results with $\alpha = 6$ are the worst and the best results are obtained with $\alpha = 3$ and 4.

From these experiments, we decided to set $\alpha = 4$ since it obtained good results and also because we would like to keep a low number of intervals in the network.

[1] We consider a TNBN containing at most 5 nodes to be small, a TNBN containing between 5 and 10 nodes to be medium and more than 10 nodes is considered large.

Table 6
Results obtained for the TNBN of Fig. 5(a) using data generated by a Gaussian and uniform distribution. The average result of varying the number of cases from 300 to 1000 is presented. For each row, the average of varying the number of initial intervals from 2 to 4 is shown. Each experiment was repeated five times. E+ is the number of added edges, E− is the number of deleted edges, BS is Brier Score, RTE is relative temporal error, SS is structural similarity, PTE is the predictive temporal error and #Int is the total number of intervals in the TNBN. An '∗' is shown if the result of LIPS is significantly better than EWD; '†', if LIPS is significantly better than *K-means* and '§', if LIPS is significantly better than Friedman's algorithm.

Algorithm	E+	E−	SS	BS	RTE	PTE	#Int
Gaussian							
EWD	0.15	1.50	0.700 ± 0.100	0.220 ± 0.015	0.307 ± 0.009	0.264 ± 0.010	6.00
K-M	0.03	1.50	0.700 ± 0.115	0.256 ± 0.011	0.317 ± 0.016	0.286 ± 0.010	6.00
F-G	0.33	1.41	0.717 ± 0.115	**0.200** ± **0.013**	0.335 ± 0.024	0.268 ± 0.014	**3.73**
LIPS	**0.00**§	**1.33**∗	**0.733** ± **0.077**	0.224 ± 0.004	**0.285** ± **0.012§**	**0.255** ± **0.005**†	5.81
Uniform							
EWD	0.16	1.50	0.700 ± 0.067	0.252 ± 0.006	0.325 ± 0.012	0.288 ± 0.008	6.00
K-M	0.24	1.41	0.717±0.084	0.256 ± 0.008	0.335 ± 0.017	0.296 ± 0.012	6.00
F-G	0.24	**1.25**	**0.750** ± **0.100**	0.239 ± 0.015	0.317 ± 0.012	**0.268** ± **0.015**	**4.37**
LIPS	**0.01**∗†§	**1.25**	**0.750** ± **0.110**∗	**0.230** ± **0.004**†	**0.303** ± **0.005**†	**0.268** ± **0.006**†	4.45

6.2. Evaluating LIPS algorithm

For all the experiments, the following methodology was applied. Data was generated according to a reference TNBN, the temporal data was generated using Gaussian and uniform distributions. Four different algorithms were evaluated in the experiments:

1. Equal-Width Discretization (EWD). This algorithm is considered a baseline. It only uses an equal width discretization to obtain intervals that correspond to the temporal nodes. It has as parameter the number of intervals to be generated.
2. *K-means* (K-M). This algorithm is also considered a baseline. It applies the K-means algorithm over the temporal data. Then, it applies an algorithm to transform the centroids into intervals for the temporal nodes. The algorithm has as parameter the number of clusters to obtain.
3. The algorithm presented in [9](F-G). This algorithm performs a discretization of the continuous data while learning the structure. This algorithm was described in Section 3.
4. Our proposed algorithm (LIPS).

For the experiments we vary the number of cases. Each experiment was repeated five times. In the tables, the best results are presented in **bold** type. We applied statistical significance tests using the Kruskal–Wallis method with significance level $\alpha_{kw} = 0.05$. In the tables we show an '∗', if the result of LIPS is significantly better than EWD, a '†' if LIPS is significantly better than *K-means* and a '§' if LIPS is significantly better than Friedman's algorithm.

6.3. Small TNBN

The first set of experiments was performed with the TNBN presented in Fig. 5(a). The objective of this experiment is to evaluate the algorithm on a very simple model.

In Table 6 the results of the experiments using a Gaussian and an Uniform distribution for obtaining the temporal data are given. Each table presents the average results varying the number of cases using 300, 400, 500 and 1000 data cases. For each number of cases 4 rows are presented that correspond to the four algorithms being compared. Each row contains the results of the seven measures presented before. For the structural similarity, Brier score, relative temporal error, and predictive temporal error, the standard deviations are also presented. Some conclusions can be obtained from these results:

- LIPS obtained in average better scores than the two baseline algorithms.
- LIPS algorithm obtained the best score in predictive temporal error with both distributions.
- The Friedman's algorithm obtained the lowest number of intervals in all the experiments in both distributions.
- For structural similarity LIPS and Friedman's algorithms obtained the best results. However, LIPS algorithm obtained a lower number of added edges.

Additionally, in Table 7 a different view of the experiments is presented. In this case, the results are presented by initialization. Column #I-I represents the number of initial intervals for the EWD, F-G, and LIPS algorithms. It represents the number of initial clusters for the K-M approximation. The results presented are the average of varying the number of data using 300, 400, 500 and 1000 instances of data.

From the results, we can observe that for all the initializations LIPS and Friedman's algorithms obtained better results than the baseline algorithms. We can also observe that the results for the K-means approximation and EWD depend on the initializations since a lower number of intervals yields to better results than an initialization with a larger number of initial intervals. This may happen due to the fact that the less intervals obtained, the larger they will be and therefore they could

Table 7
Results of the TNBN presented in Fig. 5(a) varying the number of initial intervals (#I-I).

#I-I	Algorithm	E+	E−	SS	BS	TRE	PTE	#Int
Uniform								
2	EWD	0.40	1.64	**0.750**	0.226	0.302	0.264	**4.0**
	K-M	**0.00**	1.76	0.700	**0.222**	0.331	0.277	**4.0**
	F-G	0.20	1.72	**0.750**	0.234	0.307	**0.263**	**4.0**
	LIPS	**0.00**	**1.36**	**0.750**	0.231	**0.301**	0.267	5.9
3	EWD	**0.00**	1.80	0.700	0.259	0.338	0.298	6.0
	K-M	**0.00**	1.60	0.750	0.241	0.325	0.269	6.0
	F-G	0.60	1.72	0.750	0.230	0.303	0.268	**3.6**
	LIPS	**0.00**	**1.44**	**0.800**	**0.228**	**0.301**	**0.264**	5.7
4	EWD	0.04	1.80	0.650	0.271	0.335	0.303	8.0
	K-M	0.08	1.68	0.700	0.280	0.374	0.327	8.0
	F-G	0.20	1.60	**0.750**	0.241	0.320	**0.270**	**3.6**
	LIPS	**0.00**	**1.36**	0.700	**0.231**	**0.307**	0.272	5.7
Gaussian								
2	EWD	0.08	1.44	**0.700**	**0.195**	**0.289**	**0.242**	**4.0**
	K-M	**0.00**	1.60	**0.700**	0.215	0.292	0.253	**4.0**
	F-G	0.20	1.60	**0.700**	0.204	0.348	0.276	4.2
	LIPS	0.04	**1.20**	**0.700**	0.225	0.291	0.258	4.6
3	EWD	0.20	1.84	0.700	0.214	0.298	0.256	6.0
	K-M	0.40	1.48	0.700	**0.204**	0.347	0.276	6.0
	F-G	0.20	**1.60**	**0.750**	0.224	0.285	0.255	**4.4**
	LIPS	**0.00**	**1.60**	**0.750**	0.221	**0.275**	**0.248**	**4.4**
4	EWD	0.20	1.84	0.700	0.250	0.334	0.292	8.0
	K-M	0.32	1.60	0.700	0.286	0.333	0.310	8.0
	F-G	0.32	**1.40**	0.700	**0.193**	0.311	**0.252**	4.6
	LIPS	**0.00**	**1.40**	**0.750**	0.225	**0.290**	0.257	**4.4**

Table 8
Results obtained for the TNBN of Fig. 5(b) using data generated by a Gaussian and uniform distribution. The average result of varying the number of cases with 300, 400, 500 to 1000 data instances is presented. For each row, the average of varying the number of initial intervals from 2 to 4 is shown. Each experiment was repeated five times.

Algorithm	E+	E−	SS	BS	RTE	PTE	#Int
Gaussian							
EWD	0.57	1.41	0.823 ± 0.071	0.223 ± 0.010	0.312 ± 0.004	0.261 ± 0.006	15.00
K-M	0.48	1.24	0.844 ± 0.092	0.243 ± 0.007	0.326 ± 0.010	0.285 ± 0.007	15.00
F-G	0.33	**0.41**	**0.948 ± 0.040**	**0.195 ± 0.003**	0.383 ± 0.004	0.289 ± 0.003	**10.67**
LIPS	**0.24**∗	1.25∗	0.875 ± 0.059	0.208 ± 0.006†	**0.304 ± 0.011**§	**0.257** ± †**0.009**	13.67
Uniform							
EWD	0.63	0.75	0.906 ± 0.100	0.23 ± 0.006	0.300 ± 0.012	0.264 ± 0.008	15.00
K-M	0.71	0.75	0.906 ± 0.067	0.249 ± 0.008	0.300 ± 0.017	0.275 ± 0.012	15.00
F-G	0.24	0.66	0.917 ± 0.084	**0.201 ± 0.015**	0.320 ± 0.012	0.262 ± 0.015	**10.00**
LIPS	**0.23**∗†	**0.58**	**0.927 ± 0.114**	**0.201 ± 0.004**†	**0.286 ± 0.005**	**0.244 ± 0.006**†	13.47

obtain better results in predictive score. It is important to note that LIPS algorithm is not significantly affected by varying the number of initial intervals.

6.4. Medium TNBN

The second set of experiments was performed using data obtained from the TNBN presented in Fig. 5(b). This TNBN is an extended model of the one presented in Fig. 5(a). It contains five temporal nodes. It is important to notice that in this example there are temporal nodes that have as parent only temporal nodes, therefore we will use the top-down approach proposed by LIPS.

In Table 8 the average results of the experiments are presented. Some conclusions can be obtained:

- LIPS algorithm obtained the best scores in PTE in both types of data (Gaussian and uniform).
- For the Gaussian distribution, Friedman's algorithm obtained the best score in structural similarity. However, for the uniform distribution the results obtained for LIPS and Friedman are almost the same.
- For the Gaussian distribution the results in structural similarity obtained by the K-means approximation and EWD are worse than the ones obtained with the uniform distribution.
- For both distributions, Friedman's algorithm obtained the lowest number of intervals.

Table 9
Results of the TNBN presented in Fig. 5(b) varying the number of initial intervals (#I-I).

#I-I	Algorithm	E+	E−	SS	BS	TRE	PTE	#Int
Uniform								
2	EWD	0.40	**0.60**	0.906	**0.194**	**0.286**	**0.240**	**10.00**
	K-M	0.44	0.68	0.906	0.197	0.287	0.242	**10.00**
	F-G	**0.20**	0.84	0.906	0.196	0.320	0.258	10.80
	LIPS	**0.20**	0.84	**0.938**	0.201	0.288	0.245	12.72
3	EWD	0.64	1.20	0.906	0.227	0.296	0.262	15.00
	K-M	0.48	1.08	0.906	**0.197**	0.320	0.258	15.00
	F-G	0.40	**0.84**	**0.938**	0.201	0.286	0.244	**10.20**
	LIPS	**0.20**	**0.84**	**0.938**	**0.197**	**0.285**	**0.241**	13.28
4	EWD	0.68	2.20	**0.906**	0.262	0.302	0.282	20.00
	K-M	0.52	1.60	**0.906**	0.291	0.310	0.301	20.00
	F-G	0.40	**0.80**	**0.906**	**0.205**	0.320	0.270	**11.00**
	LIPS	**0.32**	0.88	**0.906**	**0.205**	**0.285**	**0.245**	15.00
Gaussian								
2	EWD	0.56	0.44	0.875	0.200	0.307	**0.234**	**10.00**
	K-M	0.60	0.44	0.875	0.195	0.317	0.256	**10.00**
	F-G	**0.24**	**0.40**	**0.938**	**0.188**	0.381	0.285	**10.00**
	LIPS	**0.24**	0.60	0.906	0.207	**0.300**	0.254	13.20
3	EWD	0.48	0.92	0.844	0.215	0.319	0.267	15.00
	K-M	0.68	0.88	0.844	**0.197**	0.392	0.295	15.00
	F-G	**0.24**	**0.40**	**0.938**	0.208	**0.304**	**0.257**	**10.00**
	LIPS	**0.24**	0.60	0.844	0.206	0.308	0.260	13.20
4	EWD	0.84	1.60	0.750	0.252	0.310	0.281	20.00
	K-M	0.84	1.20	0.813	0.296	0.328	0.312	20.00
	F-G	0.24	**0.40**	**0.969**	**0.200**	0.374	0.287	**10.00**
	LIPS	**0.20**	0.60	0.875	0.210	**0.304**	**0.257**	14.00

- LIPS algorithm obtained the best scores in RTE and a competitive BS. In contrast, Friedman's algorithm obtained larger RTE that yielded in a larger PTE.

Table 9 presents the average results by number of initial intervals. Each row presents the average of varying the number of cases with 300, 400, 500 and 1000 instances of data.

From the results we can observe that the structural measures are not affected by different initializations for the K-mean approximation and EWD. However, the predictive score and temporal error depend on this initialization. Both measures get worse scores with a larger number of initial intervals. For LIPS and Friedman's algorithms this does not happen. LIPS obtained a lower temporal error, on the other hand, Friedman's algorithm obtained a lower number of intervals. We can conclude that for this TNBN both LIPS and Friedman are stable algorithms that do not considerably vary their results with different initializations.

6.5. Large TNBN

The third set of experiments was performed using data generated with the TNBN presented in Fig. 5(c). This TNBN was presented in [10] and was used to predict and diagnose faults in a fossil power plant. It is a TNBN used in a real world domain with a more complex structure that contains 10 temporal nodes and 4 instantaneous nodes. For this reason, the number of cases in the experiments started with 500 and was increased until 3000.

From the results presented in Table 10 we can confirm the conclusions obtained with the two smaller TNBNs:

- LIPS obtained the best score in RTE and TPE.
- Friedman's algorithm obtained the best result in BS and a lower number of intervals.
- Friedman's algorithm also obtained a larger TPE that affected negatively the PTE measure.
- LIPS and Friedman's algorithms obtained better results than the K-means approximation and EWD.
- Friedman'a algorithm obtained, in average, better results in structural similarity and LIPS obtained a lower number of added edges.

In Table 11 the results are presented by number of initial intervals. For each one, the average of the results varying the number of cases from 500 to 3000 are presented. In these results the best scores in structural similarity and predictive accuracy were obtained by Friedman's algorithm. However, LIPS obtained competitive results in Brier score and a lower relative temporal error, thus obtaining the best predictive temporal error in all the different initializations.

Table 10
Results obtained for the TNBN of Fig. 5(c) using data generated by a Gaussian and uniform distribution. The average result of varying the number of cases from 500 to 3000 is presented. For each row, the average of varying the number of initial intervals from 2 to 4 is shown. Each experiment was repeated five times.

Algorithm	E+	E−	SS	BS	RTE	PTE	#Int
Gaussian							
EWD	0.55	12.91	0.193 ± 0.080	0.147 ± 0.007	0.301 ± 0.002	0.224 ± 0.003	30.00
K-M	0.56	12.41	0.224 ± 0.116	0.230 ± 0.006	0.318 ± 0.012	0.274 ± 0.008	30.00
F-G	0.60	11.99	**0.250 ± 0.050**	**0.095 ± 0.012**	0.373 ± 0.023	0.233 ± 0.009	**20.00**
LIPS	**0.41**	**12.33**∗	0.229 ± 0.114∗	0.145 ± 0.003	**0.285 ± 0.002**†§	**0.215 ± 0.001**†	28.24
Uniform							
EWD	0.62	11.66	0.271 ± 0.149	0.205 ± 0.007	0.296 ± 0.002	0.250 ± 0.004	30.00
K-M	0.87	11.66	0.271 ± 0.149	0.249 ± 0.011	0.296 ± 0.004	0.273 ± 0.004	30.00
F-G	0.60	**11.00**	**0.312 ± 0.165**	**0.143 ± 0.011**	0.322 ± 0.006	**0.232 ± 0.006**	**20.00**
LIPS	**0.33**†	11.16	0.302 ± 0.175∗†	0.197 ± 0.006†	**0.278 ± 0.002**§	0.238 ± 0.004†	25.20

Table 11
Results of the TNBN presented in Fig. 5(c) varying the number of initial intervals (#I-I).

#I-I	Algorithm	E+	E−	SS	BS	TRE	PTE	#Int
Uniform								
2	EWD	1.28	11.20	0.266	**0.140**	0.284	**0.212**	20.0
	K-M	0.80	11.60	0.250	0.168	0.283	0.225	20.0
	F-G	0.80	**10.80**	**0.344**	0.145	0.316	0.230	**20.0**
	LIPS	**0.60**	12.00	0.328	0.178	**0.277**	0.228	25.4
3	EWD	0.33	10.33	0.281	0.209	0.292	0.251	25.0
	K-M	0.67	10.50	0.297	**0.165**	0.328	0.246	25.0
	F-G	0.17	**9.50**	**0.313**	0.197	**0.278**	0.238	**16.6**
	LIPS	**0.00**	9.83	0.297	0.206	**0.278**	**0.242**	20.8
4	EWD	0.60	13.20	0.266	0.264	0.312	0.288	40.0
	K-M	1.00	13.40	0.266	0.327	0.310	0.318	40.0
	F-G	0.80	12.40	**0.313**	**0.150**	0.321	**0.236**	**20.0**
	LIPS	**0.40**	**11.60**	0.281	0.208	**0.279**	0.243	25.2
Uniform								
2	EWD	0.80	12.56	0.234	**0.106**	0.336	0.221	**20.0**
	K-M	0.68	12.48	**0.297**	0.146	0.340	0.243	**20.0**
	F-G	1.00	**11.80**	0.281	0.119	0.370	0.244	**20.0**
	LIPS	**0.56**	12.08	0.250	0.147	**0.287**	**0.217**	28.4
3	EWD	0.32	13.32	0.203	0.143	0.284	0.213	30.0
	K-M	0.44	12.88	0.234	**0.094**	0.354	0.224	30.0
	F-G	0.40	**12.40**	**0.266**	0.145	**0.285**	**0.215**	**20.0**
	LIPS	**0.24**	12.60	0.203	0.144	0.286	**0.215**	28.2
4	EWD	0.52	13.84	0.141	0.191	0.282	0.237	40.0
	K-M	0.56	13.28	0.203	0.308	0.297	0.302	40.0
	F-G	0.40	13.00	0.203	**0.070**	0.396	0.229	**20.0**
	LIPS	**0.44**	**12.56**	**0.234**	0.146	**0.282**	**0.214**	28.1

6.6. General results

In Tables 12 and 13 a summary of the experiments using Gaussian and uniform distributions are presented. Each row presents the average results for the TNBN used for the four algorithms evaluated.

From these tables there are some conclusions:

- LIPS obtained the best results in RTE.
- Friedman's algorithm obtained the best results in BS and a lower number of intervals.
- Since Friedman's algorithm obtained the worse results in RTE, it had a significant effect in PTE.
- LIPS obtained a competitive BS and the best RTE, therefore it obtained the best PTE.
- EWD obtained competitive results in some experiments even when it is the simplest algorithm.
- LIPS and Friedman's algorithms obtained better results than the baseline algorithms in all the experiments.
- Even when LIPS used the assumption that data is Gaussian when evaluated using a uniform distribution, it obtained competitive results, even better than the baseline algorithms.

Table 12
Averages of the results with the three reference TNBNs using a Gaussian distribution to generate the temporal data.

Experiment	Alg.	E+	E−	SS	BS	RTE	PTE	#Int
Small	EWD	0.15	1.50	0.700 ± 0.100	0.220 ± 0.015	0.307 ± 0.009	0.264 ± 0.010	6.00
	K-M	0.03	1.50	0.700 ± 0.115	0.256 ± 0.011	0.317 ± 0.016	0.286 ± 0.010	6.00
	F-G	0.33	1.41	0.717 ± 0.115	**0.200 ± 0.013**	0.335 ± 0.024	0.268 ± 0.014	**3.73**
	LIPS	**0.00**§	**1.33**∗	**0.733 ± 0.077**	0.224 ± 0.004	**0.285 ± 0.012**§	**0.255 ± 0.005**†	5.81
Medium	EWD	0.57	1.41	0.823 ± 0.071	0.223 ± 0.010	0.312 ± 0.004	0.261 ± 0.006	15.00
	K-M	0.48	1.24	0.844 ± 0.092	0.243 ± 0.007	0.326 ± 0.010	0.285 ± 0.007	15.00
	F-G	0.33	**0.41**	**0.948 ± 0.040**	**0.195 ± 0.003**	0.383 ± 0.004	0.289 ± 0.003	**10.67**
	LIPS	**0.24**∗	1.25∗	0.875 ± 0.059	0.208 ± 0.006†	**0.304 ± 0.011**§	**0.257** ± †**0.009**	13.67
Large	EWD	0.55	12.91	0.193 ± 0.080	0.147 ± 0.007	0.301 ± 0.002	0.224 ± 0.003	30.00
	K-M	0.56	12.41	0.224 ± 0.116	0.230 ± 0.006	0.318 ± 0.012	0.274 ± 0.008	30.00
	F-G	0.60	11.99	**0.250 ± 0.050**	**0.095 ± 0.012**	0.373 ± 0.023	0.233 ± 0.009	**20.00**
	LIPS	**0.41**	**12.33**∗	0.229 ± 0.114∗	0.145 ± 0.003	**0.285 ± 0.002**†§	**0.215 ± 0.001**†	28.24

Table 13
Averages of the results with the three reference TNBNs using a Uniform distribution to generate the temporal data.

Experiment	Alg.	E+	E−	SS	BS	RTE	PTE	#Int
Small	EWD	0.16	1.50	0.700 ± 0.067	0.252 ± 0.006	0.325 ± 0.012	0.288 ± 0.008	6.00
	K-M	0.24	1.41	0.717±0.084	0.256 ± 0.008	0.335 ± 0.017	0.296 ± 0.012	6.00
	F-G	0.24	**1.25**	**0.750 ± 0.100**	0.239 ± 0.015	0.317 ± 0.012	**0.268 ± 0.015**	**4.37**
	LIPS	**0.01**∗†§	**1.25**	**0.750 ± 0.110**∗	**0.230 ± 0.004**†	**0.303 ± 0.005**†	**0.268 ± 0.006**†	4.45
Medium	EWD	0.63	0.75	0.906 ± 0.100	0.23 ± 0.006	0.300 ± 0.012	0.264 ± 0.008	15.00
	K-M	0.71	0.75	0.906 ± 0.067	0.249 ± 0.008	0.300 ± 0.017	0.275 ± 0.012	15.00
	F-G	0.24	0.66	0.917 ± 0.084	**0.201 ± 0.015**	0.320 ± 0.012	0.262 ± 0.015	**10.00**
	LIPS	**0.23**∗†	**0.58**	**0.927 ± 0.114**	**0.201** ± **0.004**†	**0.286 ± 0.005**	**0.244 ± 0.006**†	13.47
Large	EWD	0.62	11.66	0.271 ± 0.149	0.205 ± 0.007	0.296 ± 0.002	0.250 ± 0.004	30.00
	K-M	0.87	11.66	0.271 ± 0.149	0.249 ± 0.011	0.296 ± 0.004	0.273 ± 0.004	30.00
	F-G	0.60	**11.00**	**0.312 ± 0.165**	**0.143 ± 0.011**	0.322 ± 0.006	**0.232 ± 0.006**	**20.00**
	LIPS	**0.33**†	11.16	0.302 ± 0.175∗†	0.197 ± 0.006†	**0.278 ± 0.002**§	0.238 ± 0.004†	25.20

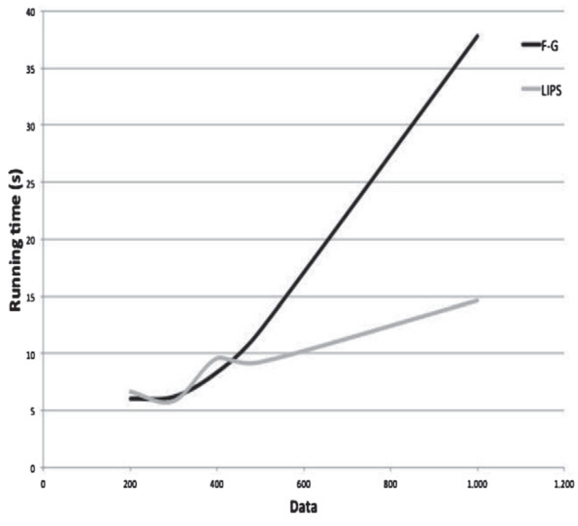
Table 14
Average and standard deviations of running times for F-G and LIPS algorithms for the three TNBNs used in the experiments. The column # data shows the number of instances used for the small and the medium TNBNs, in parenthesis is shown the number of instances used for the large TNBN. Each result is the average of 10 executions.

# Data	Algorithm	Small	Medium	Large
200 (250)	F-G	**6.04 ± 3.55**	**25.13 ± 5.10**	44.24 ± 11.28
	LIPS	6.65 ± 2.09	34.18 ± 4.09	**31.99 ± 2.55**
300 (500)	F-G	6.18 ± 2.80	**30.85 ± 3.96**	134.21 ± 57.70
	LIPS	**5.82 ± 1.07**	28.11 ± 4.02	**37.03 ± 3.86**
400 (1000)	F-G	**8.34 ± 2.72**	46.79 ± 7.93	502.30 ± 53.03
	LIPS	9.57 ± 0.80	**34.84 ± 2.58**	**65.27 ± 4.96**
500 (2000)	F-G	12.01 ± 2.33	69.16 ± 6.47	2,275.99 ± 19.17
	LIPS	**9.24 ± 0.48**	**53.64 ± 2.41**	**95.19 ± 3.81**
1000 (3000)	F-G	37.81 ± 1.68	256.69 ± 9.15	6,342.38 ± 220.22
	LIPS	**14.63 ± 1.67**	**90.60 ± 2.65**	**591.19 ± 29.43**

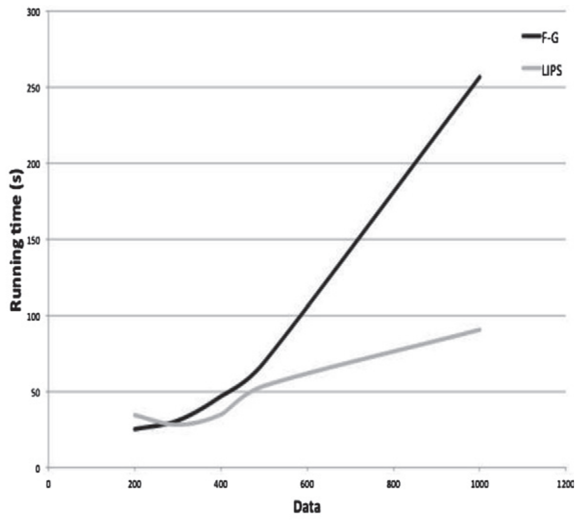
6.7. Temporal complexity experiments

In Table 14 the average and standard deviation of the running times (in seconds) of the LIPS and Friedman's algorithms are compared for different experiments varying the number of data instances for the three reference TNBNs shown in Fig. 5. From the table we can make the following observations:

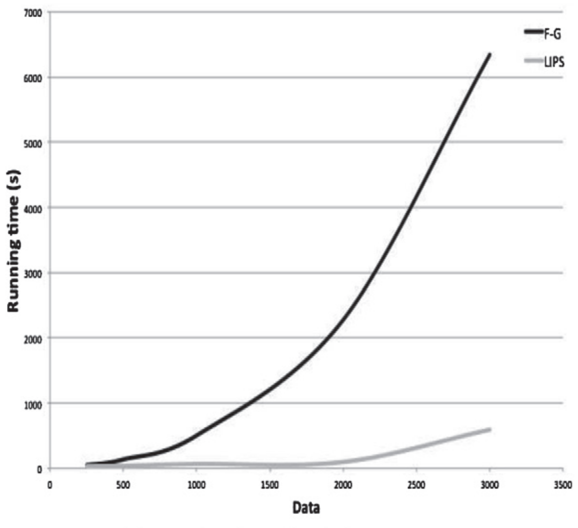
- In the small TNBN, the algorithm presented by Friedman obtained lower running times when the number of cases was less than 500, however with 500 and 1000 cases, LIPS algorithm obtained lower running times.
- In the medium TNBN it happens the same behavior, when the number of cases was small (less than 400) Friedman's algorithm obtained lower running times. As the number of cases increased, the same happens with the running times, therefore LIPS is more efficient with large number of data.
- In the large TNBN, even for a low number of cases, our algorithm obtained lower running times. Using 3000 cases for this TNBN, LIPS was approximately 20 times faster in average than the algorithm presented by Friedman.



(a) Running times for the small TNBN



(b) Running times for the medium TNBN



(c) Running times for the large TNBN

Fig. 6. Running times for the TNBNs of Fig. 5. The running times obtained by [9] are presented in black. The running times obtained by LIPS algorithm are presented in light grey.

In Fig. 6 a graph showing the running times with respect to the number of data is presented. LIPS algorithm has a lower curve in all the TNBNs than the one obtained by [9] algorithm. Some ideas that may give an intuition about the results of the algorithms are:

- Friedman's algorithm has to evaluate all the mid-points in a dataset for each node to be discretized, this number can be large depending of the dispersion on the data.
- Friedman's algorithm has a cycle that will end when the queue of nodes is empty. However, when a discretization is obtained for a node, the last step of the algorithm is to add all the nodes that are in some way connected to that node. Therefore, on strongly connected TNBNs the algorithm will have to add several nodes to the queue, increasing the running time.

7. A TNBN in a real application

Now, we present an example of a real domain application. We learned a TNBN in order to diagnose and predict temporal faults in a subsystem of a combined cycle power plant.

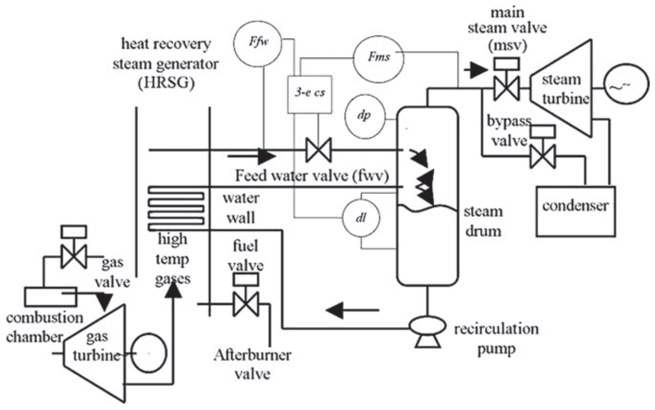


Fig. 7. Schematic description of a power plant showing the feed-water and main steam subsystems. *Ffw* refers to feedwater flow, *Fms* refers to main stream flow, *dp* refers to drum pressure, *dl* refers to drum level.

7.1. Application domain

A power plant mainly consists of three equipments: the steam generator (HRSG), the steam turbine and the electric generator. The steam generator, with the operation of burners, produces steam from the feed-water system. After the steam is superheated, it is introduced to the steam turbine to convert the energy carried out by the steam in work and finally in electricity through the corresponding steam generator. A simplified diagram of the plant is shown in Fig. 7.

The HRSG consists of a huge boiler with an array of tubes, the drum and the circulation pump. From the drum, water is supplied to the rising water tubes called water walls by means of the water recirculation pump, where it will be evaporated, and water-steam mixture reaches the drum. From here, steam is supplied to the steam turbine. The conversion of liquid water to steam is carried out at a specific saturation condition of pressure and temperature. In this condition, water and saturated steam are at the same temperature. This must be the stable condition where the volume of water supply is commanded by the feed-water control system. Furthermore, the valves that allow the steam supply to the turbine are controlled in order to manipulate the values of pressure in the drum. The level of the drum is one of the most important variables in the generation process. A decrease of the level may cause that not enough water is supplied to the rising tubes and the excess of heat and lack of cooling water may destroy the tubes. On the contrary, an excess of level in the drum may drag water as humidity in the steam provided to the turbine and cause a severe damage in the blades. In both cases, a degradation of the performance of the generation cycle is observed, and therefore this will result in loss of money for the plant.

Even with a very well calibrated instrument, controlling the level of the drum is one of the most complicated and uncertain processes of the whole generation system. In order to control the components of the power plant expert operators driving the process should be reading the levels continuously and decide the control actions that apply to this case. For this reason, it would be useful to have a model that can help the experts in order to diagnose a fault and that can show how it will affect other components. Moreover, it would be helpful to have temporal information of these events to take the best action.

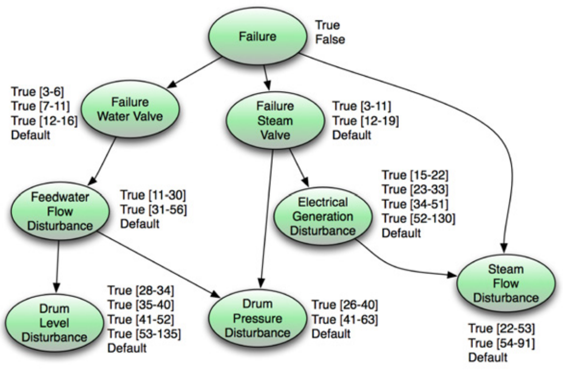
For obtaining the data used in the experiments, we used a full scale simulator of the plant. Given that the power plant contains many components, in order to simplify the problem we select some important components that would be used in the experiments, in particular:

- Feed Water Valve. We will simulate a failure in this component.
- Main Steam Valve. We will simulate a failure in this component.
- Feed-water flow. Flow of water that goes into the drum.
- Steam flow. Flow of steam that goes into the turbine.
- Drum pressure. Important value of the drum that has to be controlled.
- Drum level. Important value of the drum that has to be controlled.
- Electrical generation. Value that should be at optimal conditions.

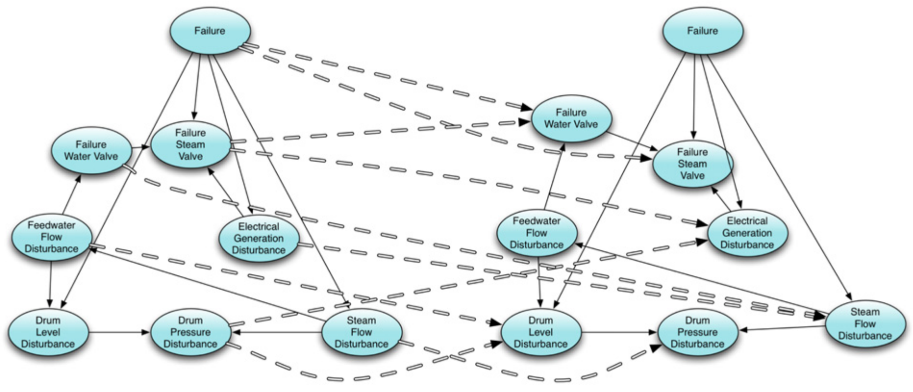
We simulate two failures randomly: failure in the Water Valve and failure in the Steam Valve. These types of failures are important because they may cause disturbances in the generation capacity and the drum. In the process, a signal exceeding its specified limit of normal functioning is called an event.

7.2. Learning a TNBN

In order to evaluate our algorithm, we obtained the structure and the intervals for each Temporal Node with the proposed algorithm. In this case, we do not have a reference network. To compare our method, we used the same three algorithms used in previous experiments: equal-width discretization, K-means clustering approximation and [9] algorithm. We evaluated the



(a) A learned TNBN



(b) A learned DBN

Fig. 8. (a) The learned TNBN for a subsystem of a combined cycle power plant. For each node the obtained temporal intervals are shown. The TNBN presents the possible effects of the failure of two valves over different important components. (b) The learned DBN for the same domain. Dashed arrows represent temporal transitions. Persistence arcs (between the same variable in different time slices) are not shown for simplicity.

Table 15
Evaluation on the power plant domain. We compare LIPS, Friedman's algorithm, K-means clustering approximation and equal-width discretization (EWD) in terms of predictive accuracy (BS), temporal error and number of intervals generated.

Num. of cases	Algorithm	BS	Temporal error	Average num. intervals
75	LIPS	**0.063**	**18.1**	16
75	EWD	0.131	19.5	24.5
75	K-M	0.143	19.3	24.5
75	F-G	0.065	27.5	**14**
150	LIPS	0.067	**17.9**	17
150	EWD	0.081	18.5	24.5
150	K-M	0.096	18.9	24.5
150	F-G	**0.066**	26.7	**14**

model using three measures: (i) the predictive accuracy using BS, (ii) the temporal error defined as the difference between the real event and the expected mean of the interval, and (iii) the number of intervals in the network. The best network should have low predictive error (BS), low temporal error and low complexity (reduced number of intervals).

We performed two experiments varying the number of cases. First, we generate the data with the simulator, then we learned the structure and the intervals. Finally, we used the learned network to compare the results with the original data. The results are presented in Table 15. The network obtained with the proposed algorithm with higher accuracy is presented in Fig. 8(a).

The following observations can be obtained from these results. In all the experiments, our algorithm obtained the best score in temporal error. For the Brier score, it obtained the best result in the first experiment and the second best result with the second experiment. Friedman's algorithm obtained the lowest number of intervals and competitive scores for the predictive accuracy. However, it obtained the worst results in temporal error. The EWD and K-means approximation obtained the highest number of intervals for both experiments and also the predictive scores were far from the best scores. It is important to note that LIPS obtains competitive results in the three measures in both experiments.

7.3. Comparison with DBNs

For comparison purposes, we used the complete data (150 instances) from the simulator to learn a dynamic Bayesian network. DBNs are similar to static BNs, in this case a probabilistic model is created to represent a process at a single point in time. Multiple copies of this model are then generated for each time point or slice belonging to a temporal range of interest. Links between copies are inserted to capture temporal relations.

Learning a DBN can be seen as a two stage process. The first stage refers to the learning of the static model and is done in an identical manner as we do with classic BNs. The second stage learns the transition network, that is, the temporal relations between random variables of different time slices. For this experiment we learned a simple DBN using the same K2 algorithm used in the TNBN to generate the static network. The transition network was learned using the Kevin Murphy's Bayesian network toolbox [16] with the Bayesian information criterion to select the best parents from the previous time slice.

Fig. 8(b) shows the learned DBN as a 2-time-slice BN, solid arrows represent arcs in the same time step, dashed arrows represent temporal transition arcs. In particular, persistence arcs (between the same variable but in consecutive time steps) were omitted for readability. The learned DBN shows some common relations with the TNBN. A total of 10 temporal arcs were learned by the DBN and 9 arcs by the TNBNs, 5 of them were learned by both models. In particular the path: *failure* → *failure steam valve* → *electrical generation disturbance* → *steam flow disturbance* is present in the TNBN and the DBN. In contrast, in the TNBN the path *failure* → *failure water valve* → *feedwater flow disturbance* → *drum level disturbance* is present, however in the DBN, the path is not complete, missing the arc between *failure water valve* → *feedwater flow disturbance*. However, the DBN was able to learn an arc between *drum pressure* and *drum level*, nodes which are indeed related, but in did not appear in the TNBN.

This example shows how TNBNs can be simpler in some domains providing an easier to understand model in cases when a series of temporal events could occur. In the case of the DBNs these series of events are harder to see because of the way the model is learned and represented. Also, it is important to notice that DBNs only represent the current time step and the next one (the interval size is fixed), in contrast the TNBN represents a number of different intervals of different size. If we wanted to capture all the intervals of the TNBN we would need to repeat the structure many times producing a cluttered DBN.

8. Conclusions and future research

We developed a method for learning both, the structure and the temporal intervals for a TNBN from data. The algorithm performs three steps: the first one obtains a basic approximation to the intervals using an equal width discretization algorithm or an approximation based on k-means clustering. The second step obtains a structure of the TNBN using the intervals obtained in the first step. The algorithm to obtain the structure is the one presented in [3]. The third step performs an interval learning algorithm using the structure obtained in the previous step. For this, it initially generates a set of candidate intervals for each Temporal Node based on a Gaussian clustering algorithm. Then, the best intervals are selected based on their predictive accuracy. We evaluated our method with synthetic data and compared it with the algorithm presented in [9] and two baseline algorithms. In general, both methods obtain similar results in terms of structure and predictive accuracy. However, our algorithm is better in balancing the temporal error and the predictive score, it is also more efficient in terms of temporal complexity.

As future research we propose different ideas. On one side, we have performed some preliminary experiments about the iteration of interval learning and structural learning. In our results the algorithm obtained the same structure in small number of iterations indicating some kind of convergence. However, we would like to formalize under what circumstances convergence is guaranteed. On the other side, we would like to apply our algorithm in a medical application regarding how HIV drugs affect the appearance of resistance mutations.

Acknowledgments

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