

Homework No. 6

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Course: Fundamentals of Digital Signal Processing

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1 Computer Projects

1. Magnitude response. Compute and plot log-magnitude response for the following cases:

- (a) Zero at $z = 1$
- (b) Zero at $z = 0.96$
- (c) Zeros at $z = e^{j\frac{\pi}{4}}$, $z = e^{-j\frac{\pi}{4}}$
- (d) Poles at $z = 0.8e^{j\frac{\pi}{4}}$, $z = 0.8e^{-j\frac{\pi}{4}}$
- (e) Poles at $z = 0.8e^{j\frac{\pi}{8}}$, $z = 0.8e^{-j\frac{\pi}{8}}$

Solution: In order to compute and plot log-magnitude response we have written the following program whose inputs are two column vectors (**a**, **b**), one vector contains the filter's zeros and the other the filter's poles. This is our proposed solution as MATLAB code:

```
% Computes the polynomials for the numerator and denominator of the system
% function based on the pole-zero pattern
[num, den] = zp2tf(z, p, k);

% Computes the frequency response vector and returns the corresponding frequency vector
% Default: 512 samples
[h, w] = freqz(num, den)

disp('Coefficients for the numerator'); disp(num);
disp('Coefficients for the denominator'); disp(den);

% Plots a z-plane and places the singularities on it
subplot(3, 1, 1);
zplane(num, den);

% Computes and plots magnitude response
subplot(3, 1, 2);
plot(w/pi, abs(h)); grid;
xlabel('\omega/\pi'); ylabel('Magnitude')

% Computes and plots log magnitude response
subplot(3, 1, 3);
plot(w/pi, 20 * log10(abs(h))); grid;
xlabel('\omega/\pi'); ylabel('Magnitude, dB')
```

This program computes system function from the Pole/Zero pattern. The system function, described in terms of the coefficients in its numerator and denominator, is used next to compute the associated frequency function, which in turn is used to obtain its magnitude response. The program displays the layout of the singularities in the Z -plane, along with magnitude response and log-magnitude response, which is calculated using the expression $20 \log_{10}(|H(e^{j\omega})|)$. Figure 1 depicts the plots produced by the program.

These are the values assigned to the vectors **a** and **b** and the gain parameter for each case

(a) $z = [1];$
 $p = [0];$
 $k = 1;$

In this case we observe that magnitude response is zero at $\omega = 0$, since there is a zero exactly in the intersection of the unit circle and positive real axis. The pole at the origin does not contribute to log-magnitude response's shape.

(b) $z = [0.96];$
 $p = [0];$
 $k = 1;$

In this case $\omega = 0$ again, but the zero does not lie exactly in the unit circle but inside. This situation prevents log-magnitude response from dropping to zero when $\omega = 0$.

(c) $z = [\cos(\pi/4) + i*\sin(\pi/4); \cos(\pi/4) - i*\sin(\pi/4)];$
 $p = [0; 0];$
 $k = 1;$

This time we have two complex-conjugate zeros at $z = e^{\pm \frac{1}{4}\pi}$. Since there are two zeros lying in the unit circle we can expect that log-magnitude response drops to zero twice, this actually happens in the plot when $\omega = \pm \frac{1}{4}\pi = \pm 0.25\pi$, however in the figure only the positive range from $0 \dots \pi$ is depicted.

(d) $z = [0; 0];$
 $p = [0.8*(\cos(\pi/4) + i*\sin(\pi/4)); 0.8*(\cos(\pi/4) - i*\sin(\pi/4))];$
 $k = 1;$

Poles outside the origin produce peaks, and in this case there are two of them when $\omega = \pm \frac{1}{4}\pi = \pm 0.25\pi$. The plot only depicts one because it considers only a positive range.

(e) $z = [0; 0];$
 $p = [0.8*(\cos(\pi/8) + i*\sin(\pi/8)); 0.8*(\cos(\pi/8) - i*\sin(\pi/8))];$
 $k = 1;$

This time we have a peak again, however this one is present before when $\omega = \pm \frac{1}{8}\pi = \pm 0.125\pi$.

2. Pole/Zero pattern. For all cases in project 2.1 plot Pole/Zero pattern and the unit impulse response.

Solution: The program used to compute and plot unit-sample response is simple. At first we use the M-file `zp2tf` to obtain the coefficients of the polynomials which compose the system function associated to each Pole/Zero pattern. Next we use `impz` to compute the coefficients of the power series expansion in z^{-1} , which turn out to be the elements of the unit-sample response sequence. Then we use MATLAB M-files to plot both the Z -plane and the sequences.

```
% Computes the polynomials for the numerator and denominator of the system
% function based on the pole-zero pattern
[num, den] = zp2tf(z, p, k);

% Computes unit impulse response
[h, t] = impz(num, den);

% Displays the result
disp('Time indexes'); disp(t);
```

```

disp('Coefficients of the power series expansion'); disp(h);

% Plots a z-plane and places the singularities on it
subplot(2, 1, 1);
zplane(num, den);

% Plots unit-sample response sequence
subplot(2, 1, 2);
stem(t, h);
xlabel('n'); ylabel('h(n)')

```

The plots produced by the program are depicted in figure 2.

3. Design the filter with the next specification using intuitive method:

$$\text{Passband: } -1.5 < |H(e^{j\omega})|_{dB} \leq 0 \quad \text{for } 0 \leq \omega \leq \frac{\pi}{4}$$

$$\text{Stopband: } |H(e^{j\omega})|_{dB} < -50 \text{ dB} \quad \text{for } \frac{3}{4}\pi \leq \omega \leq \pi$$

Plot the filter structure

Solution: The procedure used to design a filter whose log-magnitude response fulfills the specification is mostly a try and error, nondeterministic process. The MATLAB program used to design the filter is the following:

```

z = [-1;
      cos(0.75*pi)+i*sin(0.75*pi);
      cos(0.75*pi)-i*sin(0.75*pi)];
p = [0.58;
      0.85*(cos(0.25*pi)+i*sin(0.25*pi));
      0.85*(cos(0.25*pi)-i*sin(0.25*pi));
      0.09*(cos(0.10*pi)+i*sin(0.10*pi));
      0.09*(cos(0.10*pi)-i*sin(0.10*pi));
      0.312*(cos(0.15*pi)+i*sin(0.15*pi));
      0.312*(cos(0.15*pi)-i*sin(0.15*pi))];

% Gain
k = 0.014;

% Computes system function from Pole/Zero pattern
[b, a] = zp2tf(z, p, k);

% Computes the frequency response vector and returns the corresponding frequency vector
[h, w] = freqz(b, a, 1024);

% Plots the poles and zeros on the Z-plane
subplot(2, 1, 1);
zplane(z, p);

% Computes and plots log-magnitude response
subplot(2, 1, 2);
plot(w/pi, 20 * log10(abs(h))); grid
axis([0 1 -100 0]);
xlabel('\omega/\pi'); ylabel('Magnitude, dB')

% Computes and shows second-order section representation

```

```
[sos, g] = zp2sos(z, p, k);

disp('Coefficients for second-order sections: '); disp(sos);
disp('Gain: '); disp(g);
```

The most relevant part of this code is the configuration of zeros and poles used as input to the `zp2tf` M-file. The sequence of steps performed to build this Zero/Pole pattern is the following:

Step 1: We place a pole in the middle of the passband ($\omega = 0$) not so close to unit circle, as well as a zero in the middle of the stopband ($\omega = \pi$) exactly in the unit circle. As a consequence of the zero log-magnitude response drops to 0 when $\omega = \pi$ and, due to the pole, magnitude response has a peak when $\omega = 0$. See figure 3-(a).

```
z = [-1];
p = [0.58];
```

Step 2: We place two complex-conjugate zeros in the unit circle at $z = e^{\pm j\frac{3}{4}\pi}$. This provokes log-magnitude response to drop to zero exactly at the stopband frequency $\omega_s = \pm\frac{3}{4}\pi = \pm 0.75\pi$. See figure 3-(b) and notice that log-magnitude response's stopband already satisfies specification, i.e. $|H(e^{j\omega})|_{dB} < -50$ dB. So we will not do any other process for stopband.

```
z = [-1;
      cos(0.75*pi)+i*sin(0.75*pi);
      cos(0.75*pi)-i*sin(0.75*pi)];
p = [0.58];
```

Step 3: Now it is time to raise log-magnitude response in order to make it to fulfill the specifications for passband. This was achieved after spending a long time trying and correcting errors. First we raise log-magnitude response at passband frequency by placing complex-conjugate poles close to unit circle at $z = 0.85e^{\pm j\frac{1}{4}\pi}$. As shown in figure 3-(c) there is a peak in log-magnitude response at $\omega_p = \pm\frac{1}{4}\pi = \pm 0.25\pi$.

```
z = [-1;
      cos(0.75*pi)+i*sin(0.75*pi);
      cos(0.75*pi)-i*sin(0.75*pi)];
p = [0.58;
      0.85*(cos(0.25*pi)+i*sin(0.25*pi));
      0.85*(cos(0.25*pi)-i*sin(0.25*pi))];
```

Step 4: Since we are still far from the specification for passband, we try to get closer to it by placing more complex-conjugate poles, this time far from the unit circle, to raise log-magnitude response even more. We found that $z = 0.09e^{\pm j\frac{1}{10}\pi}$ was a good option. See figure 3-(d).

```
z = [-1;
      cos(0.75*pi)+i*sin(0.75*pi);
      cos(0.75*pi)-i*sin(0.75*pi)];
p = [0.58;
      0.85*(cos(0.25*pi)+i*sin(0.25*pi));
      0.85*(cos(0.25*pi)-i*sin(0.25*pi));
      0.09*(cos(0.10*pi)+i*sin(0.10*pi));
      0.09*(cos(0.10*pi)-i*sin(0.10*pi))];
```

Step 5: Finally we achieve the specification for passband by placing two more complex-conjugate poles inside the passband area. These poles are located at $z = 0.312e^{\pm j\frac{2}{20}\pi}$. Figure 3-(d) shows the final shape for log-magnitude response. Since we can not confirm that passband specification is satisfied by looking at this log-magnitude response, we provide figure 4, which is a zoom of log-magnitude response in passband. Notice that we have reached the goals of the specification.

```

z = [-1;
     cos(0.75*pi)+i*sin(0.75*pi);
     cos(0.75*pi)-i*sin(0.75*pi)];
p = [0.58;
     0.85*(cos(0.25*pi)+i*sin(0.25*pi));
     0.85*(cos(0.25*pi)-i*sin(0.25*pi));
     0.09*(cos(0.10*pi)+i*sin(0.10*pi));
     0.09*(cos(0.10*pi)-i*sin(0.10*pi));
     0.312*(cos(0.15*pi)+i*sin(0.15*pi));
     0.312*(cos(0.15*pi)-i*sin(0.15*pi))];

```

We have not finished our design process yet. The last part of the former program computes the coefficients needed to implement the filter as a cascade combination of second order sections by using the `zp2sos` M-file. The values provided by this module are the following:

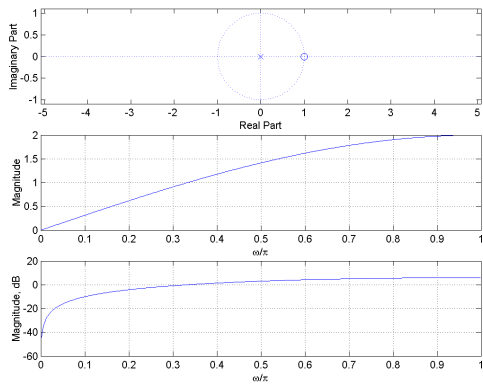
Coefficients for second-order sections:

0	1.0000	0	1.0000	-0.5800	0
0	0	1.0000	1.0000	-0.1712	0.0081
0	1.0000	1.0000	1.0000	-0.5560	0.0973
1.0000	1.4142	1.0000	1.0000	-1.2021	0.7225

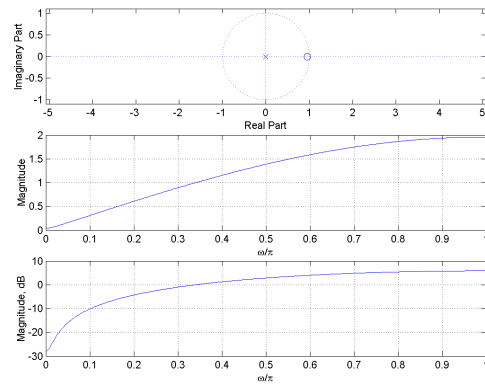
Gain:

0.0140

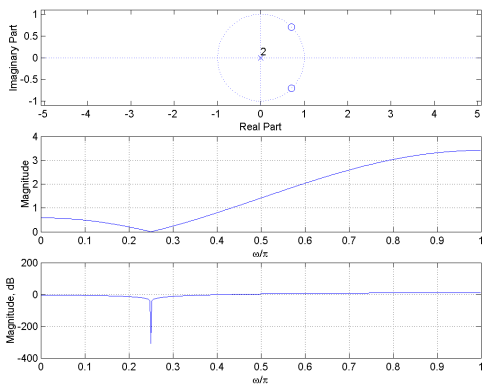
Figure 5 shows the block diagram of the implementation of the filter, which the former coefficients have been assigned to.



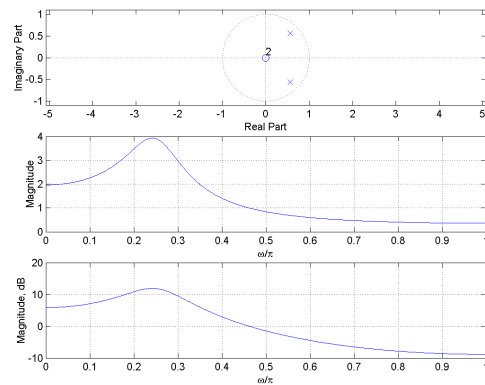
(a)



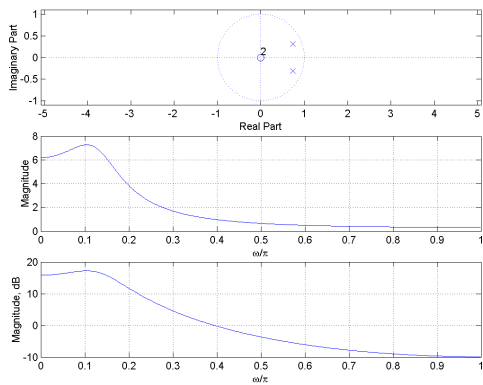
(b)



(c)

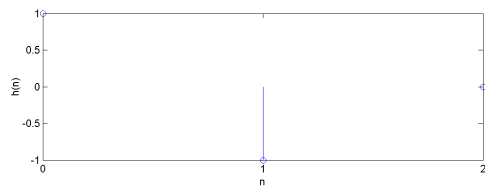
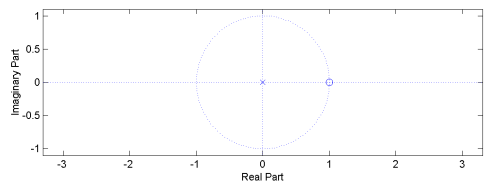


(d)

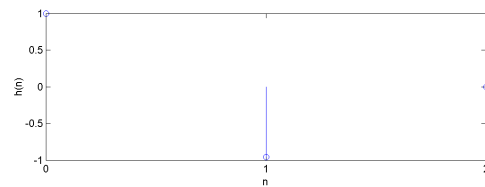
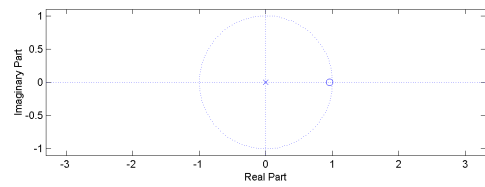


(e)

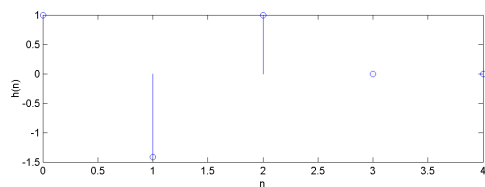
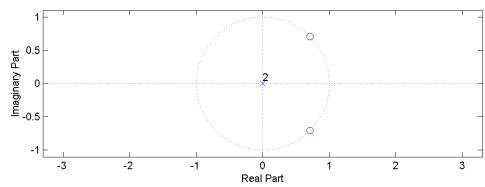
Figure 1: Pole/Zero pattern, magnitude response and log-magnitude response for data in problem 1.



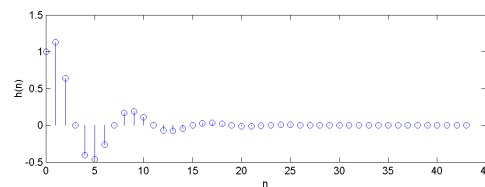
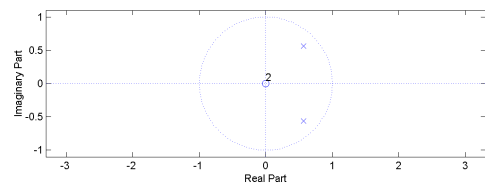
(a)



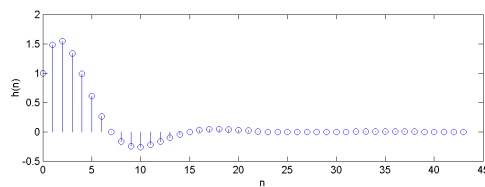
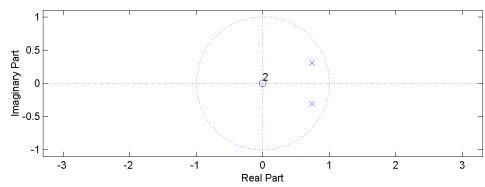
(b)



(c)

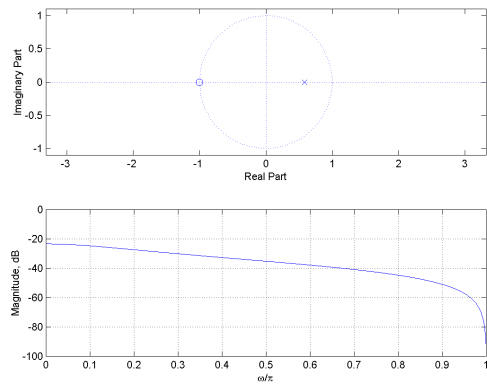


(d)

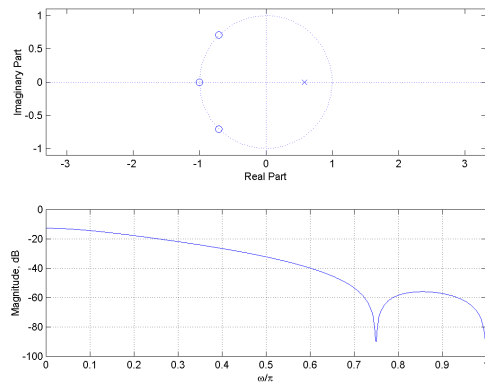


(e)

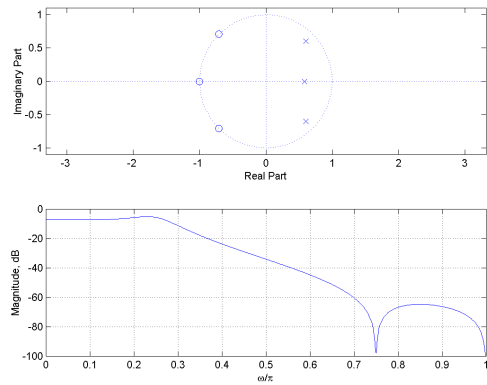
Figure 2: Pole/Zero pattern and unit-sample response for data in problem 1.



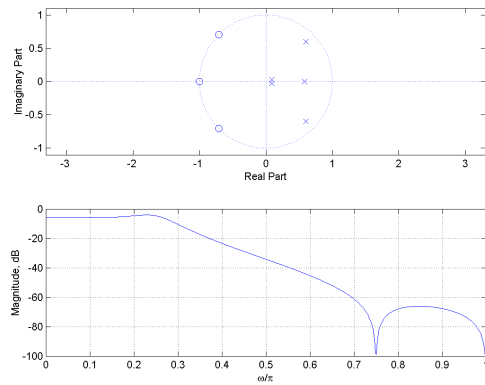
(a) Step 1



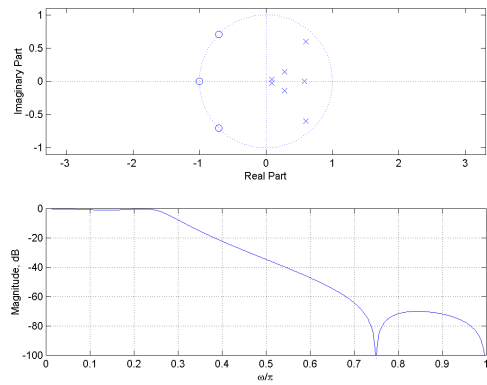
(b) Step 2



(c) Step 3



(d) Step 4



(e) Step 5

Figure 3: Designing by intuitive Pole/Zero placement.

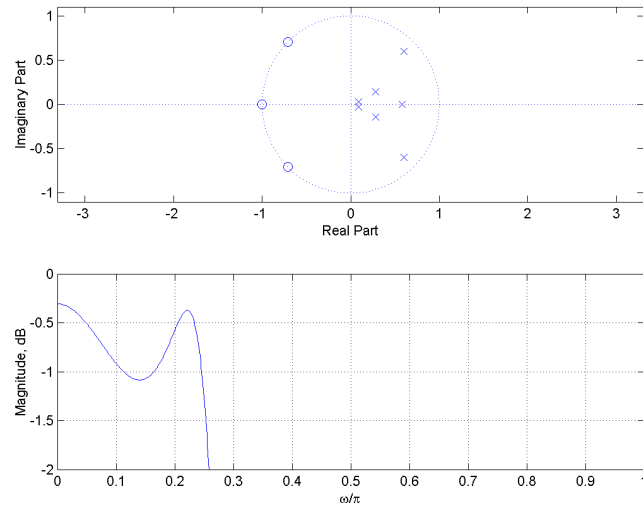


Figure 4: Zoom of log-magnitude response at passband.

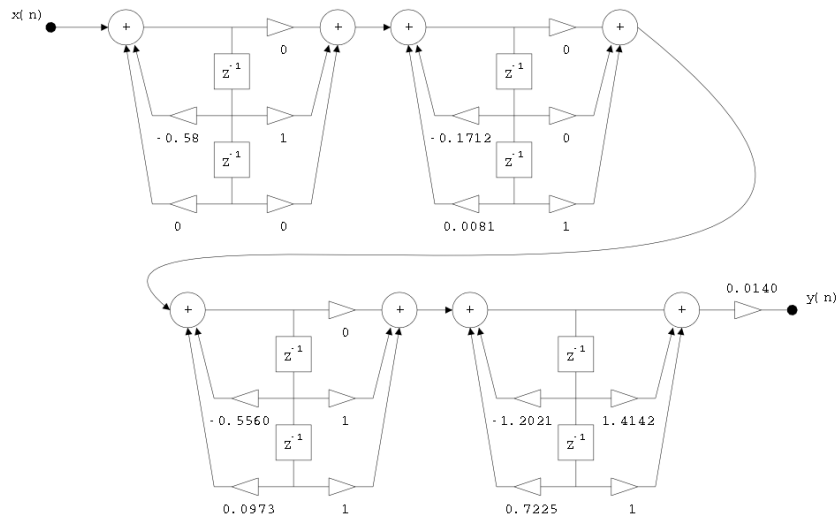


Figure 5: Filter structure as a combination of second order sections.