Homework No. 5

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Course: Fundamentals of Digital Signal Processing

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1 Problems

1. Implement the following system function in Direct Form I, Direct Form II, Cascade and Parallel structure:

$$H(z) = A\frac{1+z^{-1}}{1-0.8z^{-1}}\;\frac{1-2z^{-1}+z^{-2}}{1-1.6z^{-1}+0.8z^{-2}}$$

   Solution:

   - In order to build the implementations of the filter in Direct I and Direct II forms, we must express the system function as a rational function, i.e. the ratio of two polymonials in z^{-1}. This done next:

$$\begin{aligned} H(z) &= A\frac{1+z^{-1}}{1-0.8z^{-1}}\;\frac{1-2z^{-1}+z^{-2}}{1-1.6z^{-1}+0.8z^{-2}} \\ &= A\frac{\left(1+z^{-1}\right)\left(1-2z^{-1}+z^{-2}\right)}{\left(1-\frac{4}{5}z^{-1}\right)\left(1-\frac{8}{5}z^{-1}+\frac{4}{5}z^{-2}\right)} \\ &= A\frac{1-2z^{-1}+z^{-2}+z^{-1}-2z^{-2}+z^{-3}}{1-\frac{8}{5}z^{-1}+\frac{4}{5}z^{-2}-\frac{4}{5}z^{-1}+\frac{32}{25}z^{-2}-\frac{16}{25}z^{-3}} \\ &= A\frac{1-z^{-1}-z^{-2}+z^{-3}}{1-\frac{12}{5}z^{-1}+\frac{52}{25}z^{-2}-\frac{16}{25}z^{-3}} \qquad (1) \end{aligned}$$

     From this form we can derive block diagrams for both the Direct From I and the Direct Form II, being this last one a version which saves hardware resources such as memory. Figure 1 depicts the block diagrams of both of these implementations.

   - In order to implement the filter as a cascade combination of second order blocks, we must express its system function in the form of a product of ratios of second order polynomials in z^1. That is

$$H(z) = Az^{N_F}\prod_{k=1}^{[M/2]}\frac{1+b_{1k}z^{-1}+b_{2k}z^{-2}}{1+a_{1k}z^{-1}+a_{2k}z^{-2}}$$

     Notice that the specification of the system function we are provided with is already expressed in this form, so now it is only a matter of building the structure. The coefficients for the first second order section are taken directly from the numerator and denominator of the first factor, and the coefficients for the next second order section are taken from the numerator and denominator of the second factor. Finally we must combine these two sections in cascade to obtain the ovarall implementation. Figure 2 shows this implementation.

- Implementing the filter as a parallel combination of second order sections is not as immediate as the in former two cases. This time we must express system function as a sum of ratios of a first order polynomial and a second order polynomial.

$$H(z) = z^{N_F} \sum_{k=1}^{[M/2]} \frac{g_{0k} + g_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

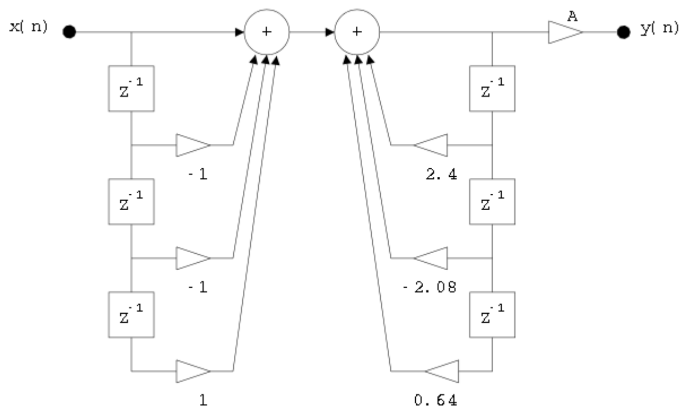
Our task is to express the rational form of the system function given in equation 1 as a sum of two ratios of polynomials. We already know the denominators of each of these two terms, they are the denominators of the original specification of the system function. Our problem reduces to determine the coefficients A, B, C and D in the following expression:

$$\frac{1 - z^{-1} - z^{-2} + z^{-3}}{\left(1 - \frac{4}{5} z^{-1}\right)\left(1 - \frac{8}{5} z^{-1} + \frac{4}{5} z^{-2}\right)} = \frac{A + B z^{-1}}{1 - \frac{4}{5} z^{-1}} + \frac{C + D z^{-1}}{1 - \frac{8}{5} z^{-1} + \frac{4}{5} z^{-2}}$$

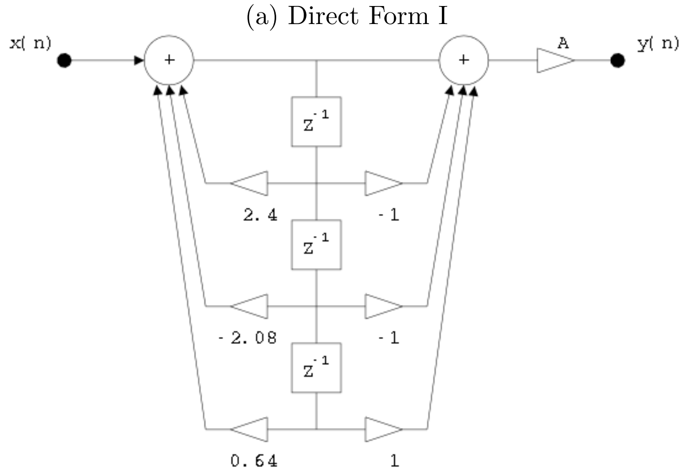
After performing some algebra we arrive to the desired expression:

$$H(z) = \frac{1 - z^{-1} - z^{-2} + z^{-3}}{\left(1 - \frac{4}{5} z^{-1}\right)\left(1 - \frac{8}{5} z^{-1} + \frac{4}{5} z^{-2}\right)} = \frac{\frac{5}{4} z^{-1} - 1}{1 - \frac{4}{5} z^{-1}} + \frac{2 - \frac{9}{4} z^{-1}}{1 - \frac{8}{5} z^{-1} + \frac{4}{5} z^{-2}}$$

So, the coefficients for the first parallel section are taken from the first term and the coefficients for the second parallel section from the second ratio of polynomials. The block diagram for this kind of implementation is illustrated in figure 3.



(a) Direct Form I



(b) Direct Form II

Figure 1: Implementations of the digital filter in Direct Form I and Direct Form II.

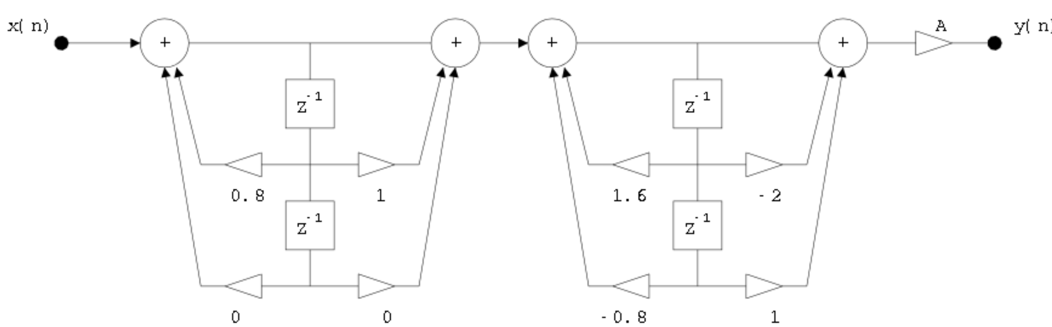


Figure 2: Implementation of the digital filter as a cascade combination of second order sections.

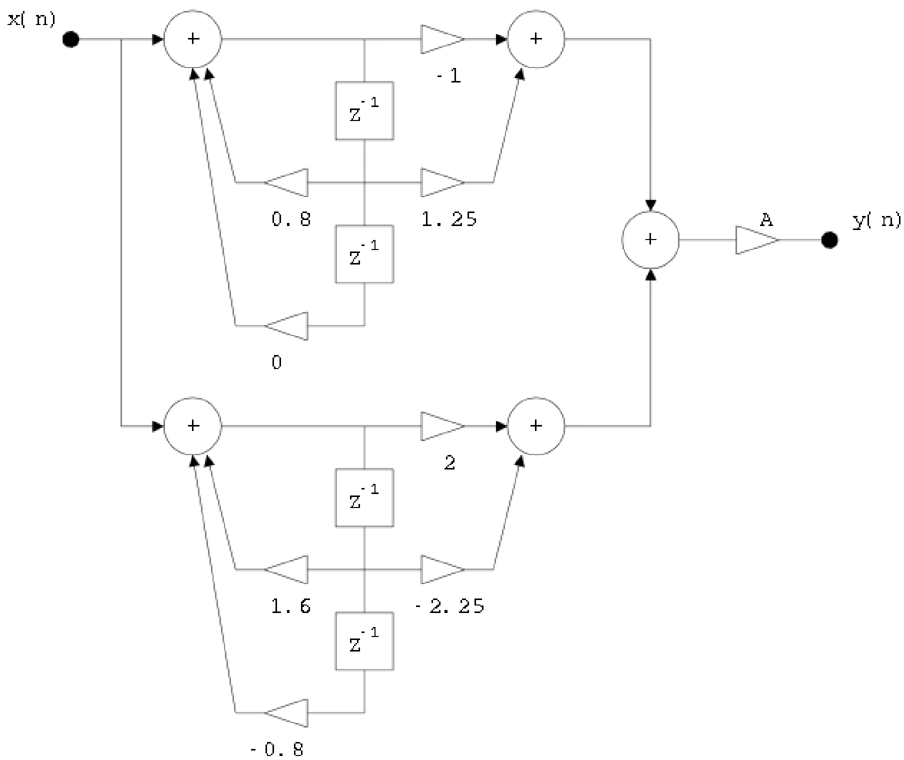


Figure 3: Implementation of the digital filter as a parallel combination of second order sections.