Homework No. 4

Tomás Balderas Contreras

Instituto Nacional de Astrofísica, Óptica y Electrónica

Course: Fundamentals of Digital Signal Processing

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1 Problems

1. Given the following unit-sample response sequences, compute Z-transform, sketch the pole/zero pattern in Z-plane and the filter block diagram:

(a) $x(n) = \begin{cases} 1 & n = -1 \\ -2 & n = 0 \\ 1 & n = 1 \\ 0 & elsewhere \end{cases}$

(b) $h(n) = \begin{cases} r^n \sin(\omega_0 n) & n \geq 0 \\ 0 & otherwise \end{cases}$, $r = 0.9$, $\omega_0 = \frac{\pi}{8}$

Solution:

(a) By using the definition of Z-transform we get the following expression:

$$
\begin{aligned}
H(z) &= \sum_{n=-\infty}^{\infty} h(n) z^{-n} \\
&= \sum_{n=-1}^{1} h(n) z^{-n} \\
&= h(-1)z + h(0) + h(1)z^{-1} \\
&= z - 2 + z^{-1} \\
&= z\left(1 - 2z^{-1} + z^{-2}\right) \\
&= z\left(1 - z^{-1}\right)^2
\end{aligned}
$$

A simple examination of the last expression shows us that there are two zeros at $z = 1$ and two poles at $z = 0$ due to the $\left(1 - z^{-1}\right)^2$ factor and one zero at $z = 0$ and one pole at $z = \infty$ due to the z factor. However, one pole and one zero at the origin cancel each other so only one pole remains there, which along with the pole at infinity preserve the property of having the same number of zeros and poles. Pole/Zero pattern for this transform is depicted in figure 1-(a), notice that MATLAB does not plot pole at $z = \infty$.

This filter's structure is shown in figure 2-(a), this block diagram was built considering the filter as a cascade combination of the filters $H_1(z) = z - 2 + z^{-1}$ and $H_2(z) = 1$.

(b) By using the definition of Z-transform and the coefficients given by $h(n)$, we get the following general expression:

$$
H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}
$$

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$$= \sum_{n=0}^{\infty} h(n)z^{-n}$$

$$= \sum_{n=0}^{\infty} r^n \sin(\omega_0 n) z^{-n}$$

$$= \sum_{n=0}^{\infty} r^n \left(\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right) z^{-n}$$

$$= \frac{1}{2j} \left(\sum_{n=0}^{\infty} \left(re^{j\omega_0} z^{-1} \right)^n - \sum_{n=0}^{\infty} \left(re^{-j\omega_0} z^{-1} \right)^n \right)$$

$$= \frac{1}{2j} \left(\frac{1}{1 - re^{j\omega_0} z^{-1}} - \frac{1}{1 - re^{-j\omega_0} z^{-1}} \right) \quad \text{since } |re^{-j\omega_0} z^{-1}| = |0.9 e^{-j\omega_0} z^{-1}| < 1$$

$$= \frac{1}{2j} \left(\frac{1 - re^{-j\omega_0} z^{-1} - 1 + re^{j\omega_0} z^{-1}}{\left(1 - re^{j\omega_0} z^{-1}\right)\left(1 - re^{-j\omega_0} z^{-1}\right)} \right)$$

$$= \frac{1}{2j} \left(\frac{r\left(e^{j\omega_0} - e^{-j\omega_0}\right) z^{-1}}{\left(1 - re^{j\omega_0} z^{-1}\right)\left(1 - re^{-j\omega_0} z^{-1}\right)} \right)$$

$$= \frac{r\sin(\omega_0)z^{-1}}{\left(1 - re^{j\omega_0} z^{-1}\right)\left(1 - re^{-j\omega_0} z^{-1}\right)}$$

$$= \frac{r\sin(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}}$$

From this general expression we realize that there is one zero at $z = \infty$ and one pole at $z = 0$ due to the factor z^{-1}. One pole exists at $z = re^{j\omega_0}$ along with the corresponding zero at $z = 0$ due to the factor $1 - re^{j\omega_0} z^{-1}$. Similarly there exists one pole at $z = re^{-j\omega_0}$ and another zero at $z = 0$ due to the factor $1 - re^{-j\omega_0} z^{-1}$. As a consequence one pole and one zero at the origin cancel each other and only one zero remains there. Notice that there are the same number of poles and zeros. The Pole/Zero pattern for this system function and the parameters $r = 0.9$, $\omega_0 = \frac{\pi}{8}$ is illustrated in figure 1-(b), again MATLAB does not indicate the existence of the singularity at $z = \infty$ but it is there.

Again we have built the second order recursive filter's block diagram as a cascade combination $\left(H(z) = H_1(z)H_2(z)\right)$ of the following filters

$$H_1(z) = r\sin(\omega_0)z^{-1} \qquad H_2(z) = \left[1 - 2r\cos(\omega_0)z^{-1} + r^2 z^{-2}\right]^{-1}$$

The structure is shown in figure 2-(b).

2. Compute Z-transform of $\{h(n)\}$, sketch the pole/zero pattern and the filter block diagram

   (a) $y(n) = x(n) - x(n - N); \; N = 5$
   (b) $y(n) = x(n) + x(n - N); \; N = 5$

Solution: In this case we are not provided with explicit values of the filters' unit-sample response sequences, however we are given the difference equation for each filter. So, in order to compute unit-sample response from these difference equations let $\{x(n)\} = \{\delta(n)\}$ and proceed with the evaluation.

   (a) In this case we have $h(n) = \delta(n) - \delta(n - 5)$. For every $n < 0$ we notice that $h(n) = 0$ since $\delta(n) = \delta(n - 5) = 0$. If $n = 0$ then $h(0) = \delta(0) - \delta(-5) = 1 - 0 = 1$. In the range $1 \le n \le 4$ it is satisfied that $h(n) = 0$. There is only one more value of n on which $h(n)$ is nonzero, i.e.
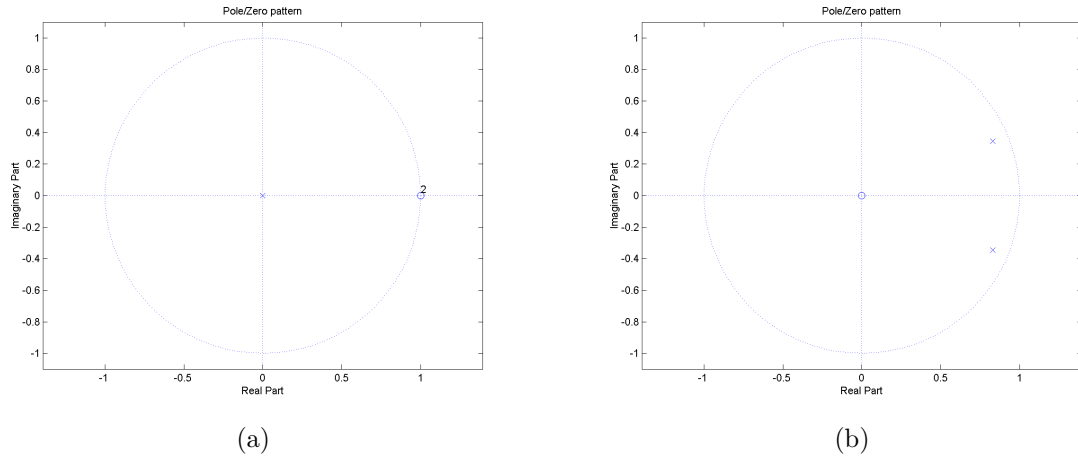
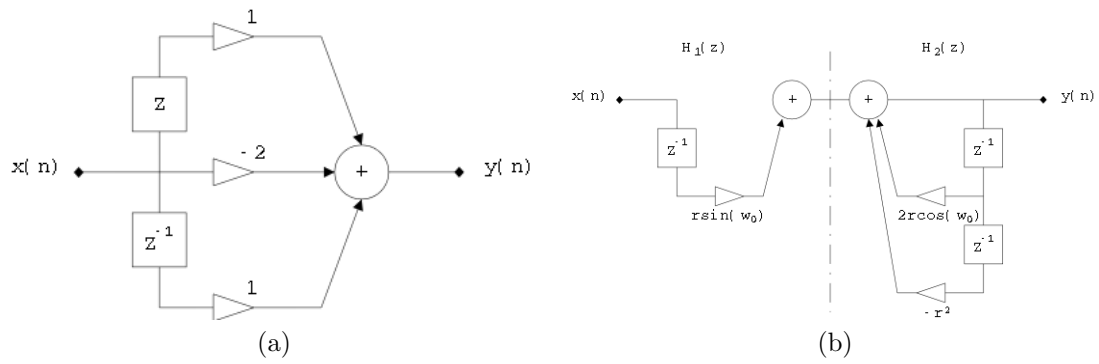Figure 1: Pole/Zero patterns for Z-transforms in problem 1.

Figure 2: Filter structures for Z-transforms in problem 1.

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$n = 5$, and as a consequence $h(5) = \delta(5) - \delta(0) = 0 - 1 = -1$. For every $n > 5$ we notice that $h(n) = 0$. Our conclusion is

$$h(n) = \begin{cases} 1 & n = 0 \\ -1 & n = 5 \\ 0 & otherwise \end{cases}$$

From here we can easily compute the filter's system function as follows:

$$
\begin{aligned}
H(z) &= \sum_{n=-\infty}^{\infty} h(n) z^{-n} \\
&= \sum_{n=0}^{5} h(n) z^{-n} \\
&= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} \\
&= 1 - z^{-5} \\
&= (1 - z^{-1})(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}) \\
&= \left(1 - z^{-1}\right)\left(1 - \frac{1}{4}\left(\sqrt{5} - 1 \pm j\sqrt{2\sqrt{5} + 10}\right)z^{-1}\right) \\
&\quad \left(1 - \frac{1}{4}\left(-\sqrt{5} - 1 \pm j\sqrt{10 - 2\sqrt{5}}\right)z^{-1}\right)
\end{aligned}
$$

After performing some algebra we have found the singularities of the system function, which turn out to be only zeros, five of them actually. All the zeros are located on the unit circle and there is a pole at $z = 0$ for each of them as well. So the property of equality among the number of zeros and poles is held. Figure 3-(a) depicts the Pole/Zero pattern for this filter and figure 4-(a) illustrates its structure, this filter is stable, causal and non-recursive.

(b) Using the same reasoning as in the last example we can determine the unit-sample response sequence from the filter's difference equation. This time we arrive to the following result:

$$h(n) = \begin{cases} 1 & n = 0 \\ 1 & n = 5 \\ 0 & otherwise \end{cases}$$

Now we perform the computations required to obtain the system function as follows:

$$
\begin{aligned}
H(z) &= \sum_{n=-\infty}^{\infty} h(n) z^{-n} \\
&= \sum_{n=0}^{5} h(n) z^{-n} \\
&= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} \\
&= 1 + z^{-5} \\
&= (1 + z^{-1})(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}) \\
&= \left(1 + z^{-1}\right)\left(1 - \frac{1}{4}\left(1 + \sqrt{5} \pm j\sqrt{10 - 2\sqrt{5}}\right)z^{-1}\right) \\
&\quad \left(1 - \frac{1}{4}\left(1 - \sqrt{5} \pm j\sqrt{2\sqrt{5} + 10}\right)z^{-1}\right)
\end{aligned}
$$

Again, the five singularities found are zeros. These zeros are exactly on the unit circle and each of them is asociated with a pole at $z = 0$ which results in five poles at the origin. This
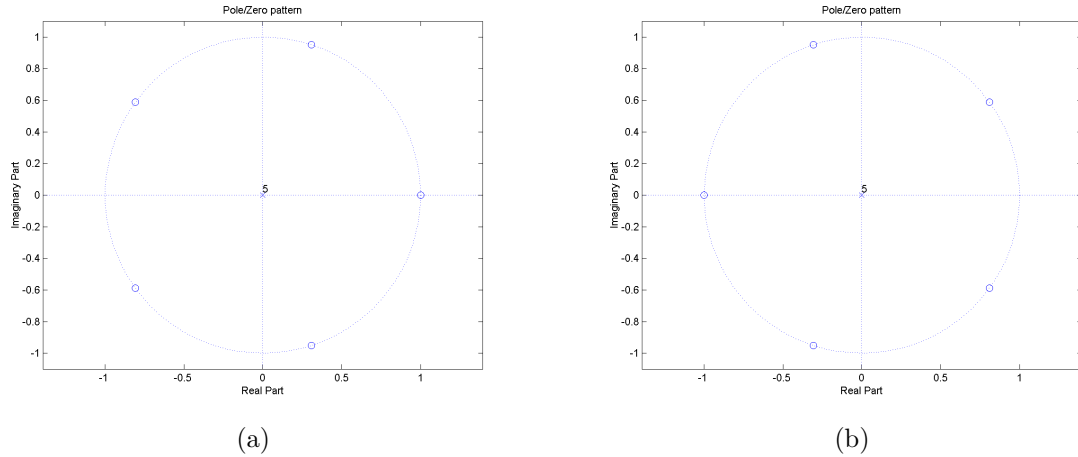
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Figure 3: Pole/Zero patterns for Z-transforms in problem 2.

Pole/Zero pattern preserves the property of equality of the number of zeros and poles. Figure 3-(b) shows the layout of the singularities on the Z-plane. This is a causal, non-recursive, stable filter whose block diagram is illustrated in figure 4-(b).

3. Compute Z-transform, sketch the filter block diagram and sketch the unit-sample response if the filter gain $A = 1$, and:

   (a) Two zeros in $z = 0$, one pole at $z = a$ and one pole at $z = -a$ where $a < 1$
   (b) Two poles in $z = 0$ and two zeros in $z = \pm jb$, where $b < 1$

Solution:

   (a) From the description of the Pole/Zero pattern given above we can conclude several facts about the system function and the filter. These are our conclusions:

   - ROC is outside the pole having the largest radius, i.e. $z = a$ (or $z = -a$)
   - The unit-sample response corresponding to this filter has infinite duration since there are finite poles located in the region $0 < |z| < \infty$
   - The filter is stable since it contains poles which are inside the unit circle
   - It is a causal system since there are no poles at $z = \infty$
   - This is a recursive filter since there are poles that do not lie at the origin

   According to the description of the system function we conclude that its numerator equals one and its denominator is a product of two terms. Next, to compute unit-sample response we perform a long division of the numerator by the denominator of the system function and get the following:

$$
\begin{aligned}
H(z) &= \frac{1}{\left(1 + az^{-1}\right)\left(1 - az^{-1}\right)} \\
&= \frac{1}{1 + az^{-1} - az^{-1} - a^2 z^{-2}} \\
&= \frac{1}{1 - a^2 z^{-2}} \\
&= \frac{z^2}{z^2 - a^2} \\
&= 1 + a^2 z^{-2} + a^4 z^{-4} + a^6 z^{-6} + a^8 z^{-8} + a^{10} z^{-10} + \ldots
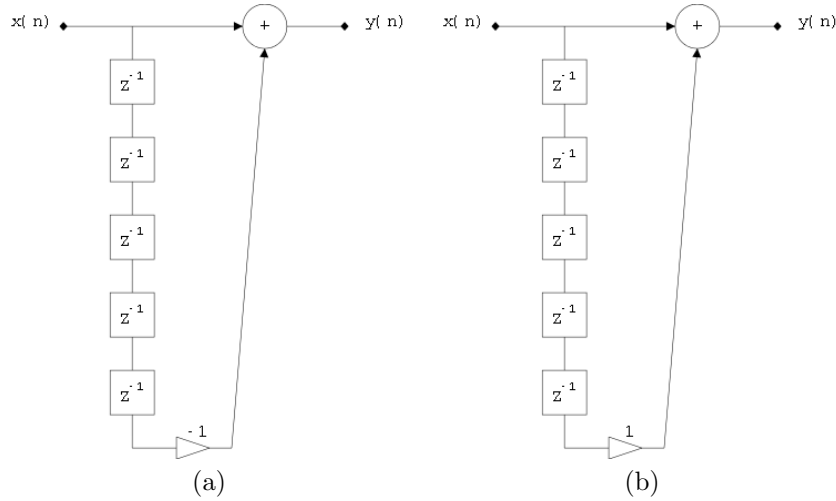\end{aligned}
$$

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Figure 4: Filter structures for Z-transforms in problem 2.

The coefficients of this polynomial in z^{-1} are the components of the unit-sample response sequence. Analizing its behavior we conclude that:

$$h(n) = \begin{cases} 1 & n = 0 \\ a^n & n > 0 \text{ and } n \text{ is even} \\ 0 & otherwise \end{cases}$$

The plot of this sequence when $a = 0.9$ is depicted in figure 5-(a) and the structure of the filter is shown in figure 6-(a).

(b) These are the comments for the filter described by the Zero/Pole pattern mentioned for this part:

- It is a causal system since there are no poles at $z = \infty$
- Unit-sample response is a finite duration sequence since the largest finite pole radius equals zero
- Since the unit-sample response sequence has finite duration the system is stable

This time we have only two poles so the resulting system function will not be a rational function but a simple polynomial in z^{-1}. Lets compute system function:

$$\begin{aligned} H(z) &= \left(1 - be^{j\frac{\pi}{2}}z^{-1}\right)\left(1 - be^{-j\frac{\pi}{2}}z^{-1}\right) \\ &= 1 - be^{j\frac{\pi}{2}}z^{-1} - be^{-j\frac{\pi}{2}}z^{-1} + b^2 z^{-2} \\ &= 1 - b\left(e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}\right)z^{-1} + b^2 z^{-2} \\ &= 1 - 2b\cos\left(\frac{\pi}{2}\right)z^{-1} + b^2 z^{-2} \\ &= 1 + b^2 z^{-2} \end{aligned}$$

The unit-sample response is totally defined by the coefficients of this result, as we predicted we got a finite duration sequence which is formally described as:

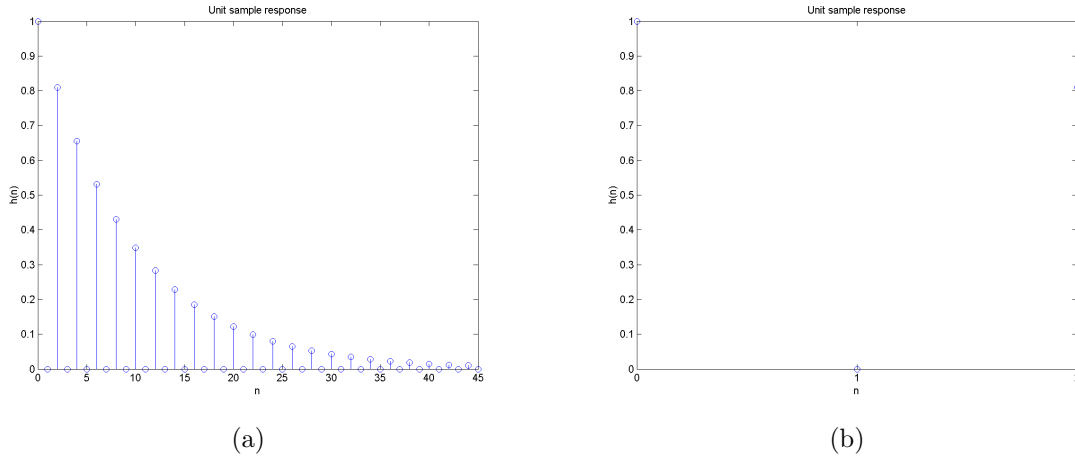$$h(n) = \begin{cases} 1 & n = 0 \\ b^2 & n = 2 \\ 0 & otherwise \end{cases}$$

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(a)   (b)

Figure 5: Unit-sample response sequences computed in problem 3.

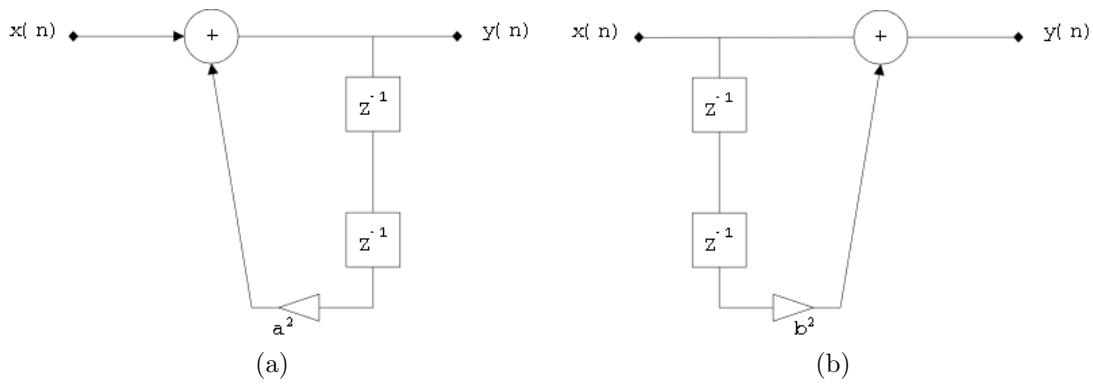(a)   (b)

Figure 6: Filter structures for Z-transforms in problem 3.

A plot of this sequence, when $b = 0.9$ is shown in figure 5-(b). The structure of the corresponding filter is shown in figure 6-(b).

4. Determine the pole locations of the system having the following unit-sample response and determine whether it is stable:

$$h(n) = \begin{cases} (-1)^n & n \geq 0 \\ 0 & otherwise \end{cases}$$

Solution: In order to find out the location of the poles of filter's system function we must, at first, compute Z-transform of unit-sample response. This is done next

$$
\begin{aligned}
H(z) &= \sum_{n=-\infty}^{\infty} h(n) z^{-n} \\
&= \sum_{n=0}^{\infty} (-1)^n z^{-n} \\
&= 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5} + z^{-6} - z^{-7} + \ldots \\
&= \sum_{n=0}^{\infty} z^{-2n} - \sum_{n=0}^{\infty} z^{-(2n+1)}
\end{aligned}
$$

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$$= \sum_{n=0}^{\infty}(z^{-2})^n - \sum_{n=0}^{\infty}z^{-2n}z^{-1}$$

$$= \sum_{n=0}^{\infty}(z^{-2})^n - z^{-1}\sum_{n=0}^{\infty}(z^{-2})^n$$

$$= \left(1 - z^{-1}\right)\sum_{n=0}^{\infty}(z^{-2})^n$$

$$= \left(1 - z^{-1}\right)\frac{1}{1 - z^{-2}} \quad |z^2| > 1$$

$$= \frac{1 - z^{-1}}{\left(1 - z^{-1}\right)\left(1 + z^{-1}\right)} \quad |z^2| > 1$$

$$= \frac{1}{1 + z^{-1}}$$

From this system function we can quickly realize that there exists only one pole at $z = -1$ and a corresponding zero at $z = 0$. Since the pole of $H(z)$ lies exactly on the unit circle we conclude that the system described by the unit-sample response provided is not stable.

5. Consider a second order filter having a finite duration unit-sample response. Let its magnitude response be equal zero at two frequency points $\omega = \omega_0$ and $\omega = -\omega_0$ whithin the range from $-\pi$ to $+\pi$.

   (a) Determine the magnitude response for all frequencies, using the gain equal 1.

   (b) Draw pole/zero pattern for the filter if the filter is causal. Is this an stable filter?

   (c) Draw filter structure for this case

Solution: The system's unit-sample response has finite duration, this implies that the filter is a nonrecursive one, which means that any pole must be found only at $z = 0$ or $z = \infty$, but since the filter is causal then there is no pole at $z = \infty$. These poles at the origin do not contribute to magnitude response but do produce linear phase. Since the magnitude response is zero when $\omega = \omega_0$ or $\omega = -\omega_0$ then we have two zeros lying on the unit circle, such zeros are $q_1 = e^{j\omega_0}$ and $q_2 = e^{-j\omega_0}$ and the poles at the origin just discussed are associated to these zeros. We have delimited the location of poles inside de unit circle, which implies that the system is stable.

To compute magnitude response we will use its definition in terms of the distances between an arbitrary point in the unit circle $(e^{j\omega})$ and the poles $(z = 0)$ and zeros $(z = e^{\pm j\omega_0})$.

$$\begin{aligned}
|H(e^{j\omega})| &= |A|\frac{D(e^{j\omega_0}, e^{j\omega})D(e^{-j\omega_0}, e^{j\omega})}{D(0, e^{j\omega})^2} \\
&= D(e^{j\omega_0}, e^{j\omega})D(e^{-j\omega_0}, e^{j\omega}) \\
&= \left(\left[\cos(\omega_0) - \cos(\omega)\right]^2 + \left[\sin(\omega_0) - \sin(\omega)\right]^2\right)^{\frac{1}{2}} \\
&\quad \left(\left[\cos(\omega_0) - \cos(\omega)\right]^2 + \left[-\sin(\omega_0) - \sin(\omega)\right]^2\right)^{\frac{1}{2}} \\
&= \left(\cos^2(\omega_0) - 2\cos(\omega_0)\cos(\omega) + \cos^2(\omega) + \sin^2(\omega_0) - 2\sin(\omega_0)\sin(\omega) + \sin^2(\omega)\right)^{\frac{1}{2}} \\
&\quad \left(\cos^2(\omega_0) - 2\cos(\omega_0)\cos(\omega) + \cos^2(\omega) + \sin^2(\omega_0) + 2\sin(\omega_0)\sin(\omega) + \sin^2(\omega)\right)^{\frac{1}{2}} \\
&= \left(2 - 2\left[\cos(\omega_0)\cos(\omega) + \sin(\omega_0)\sin(\omega)\right]\right)^{\frac{1}{2}} \\
&\quad \left(2 - 2\left[\cos(\omega_0)\cos(\omega) - \sin(\omega_0)\sin(\omega)\right]\right)^{\frac{1}{2}}
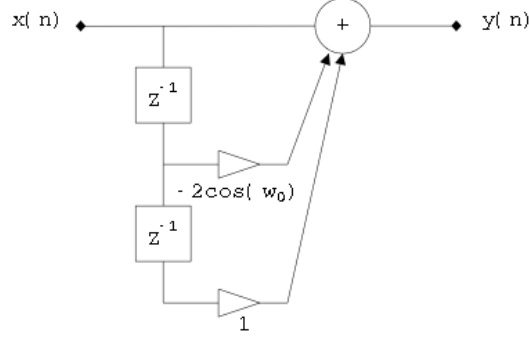\end{aligned}$$

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Figure 7: Filter structure for Z-transforms in problem 5.

$$
\begin{aligned}
&= \left(2 - 2\cos(\omega_0 - \omega)\right)^{\frac{1}{2}} \left(2 - 2\cos(\omega_0 + \omega)\right)^{\frac{1}{2}} \\
&= 2\left(1 - \cos(\omega_0 - \omega)\right)^{\frac{1}{2}} \left(1 - \cos(\omega_0 + \omega)\right)^{\frac{1}{2}}
\end{aligned}
$$

In order to build the structure of the filter we must compute system function to get the coefficients for the implementation.

$$
\begin{aligned}
H(z) &= \left(1 - e^{j\omega_0}z^{-1}\right)\left(1 - e^{-j\omega_0}z^{-1}\right) \\
&= 1 - e^{j\omega_0}z^{-1} - e^{-j\omega_0}z^{-1} + z^{-2} \\
&= 1 - \left(e^{j\omega_0} + e^{-j\omega_0}\right)z^{-1} + z^{-2} \\
&= 1 - 2\cos(\omega_0)z^{-1} + z^{-2}
\end{aligned}
$$

The block diagram of the filter is shown in figure 7.

6. Consider the filter with unit-sample response $h(n)$ and input $x(n)$, given by:

$$
h(n) = \begin{cases} 1 & n = 0 \\ -1 & n = 8 \\ 1 & n = 1 \\ 0 & otherwise \end{cases} \qquad x(n) = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases}
$$

Determine the output from the inverse Z-transform $Y(Z) = H(Z)X(Z)$.

Solution: First we will compute the system function, i.e. h sequence's Z-transform:

$$
H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} = \sum_{n=0}^{8} h(n)z^{-n} = h(0) + h(8)z^{-n} = 1 - z^{-8}
$$

Now it is time to compute x sequence's transfer function as follows:

$$
X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \sum_{n=0}^{\infty}(z^{-1})^n = \frac{1}{1 - z^{-1}} \quad |z| > 1
$$

Next we multiply these expressions to get the Z-transform of the convolution of the sequences in time domain:

$$
Y(z) = H(z)X(z) = \frac{1 - z^{-8}}{1 - z^{-1}} = \frac{-(z^{-8} - 1)}{-(z^{-1} - 1)} = \frac{z^{-8} - 1}{z^{-1} - 1}
$$

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The last step is to apply any method to perform inverse Z-transform to get the sequence $\{y(n)\}$. In this case we apply the long division method and express $Y(z)$ as a polynomial in z^{-1} whose coefficients are precisely the elements of the required sequence. Therefore

$$Y(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7}$$

Finally, we construct the desired sequence, product of the inverse Z-transform procedure:

$$y(n) = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & otherwise \end{cases}$$

7. Compute the inverse Z-transform:

$$H(Z) = \frac{1}{1 - az^{-1}} \qquad \text{ROC:} |z| > a$$

using Residue Theorem Method.

Solution: To enclose all the poles, the contour C is situated in the region of convergence. Then

$$H(z)z^{n-1} = \frac{z^n}{z - a}$$

- For $n > 0$, C encloses only one pole, at $z = a$. Then

$$P(z) = H(z)z^{n-1}(z - a) = z^n$$

  Evaluating the residue, we get

$$h(n) = P(a) = a^n \quad \forall\, n > 0$$

- For $n < 0$, C also encloses additional poles at $z = 0$. For $n = -1$:

$$H(z)z^{n-1} = \frac{1}{z(z - a)}$$

  Then,

$$h(-1) = \text{Res}[H(z)z^{n-1}, z = 0] + \text{Res}[H(z)z^{n-1}, z = a]$$

  The residue at $z = 0$ is equal to

$$P(z) = \frac{1}{z - a} \qquad P(0) = -\frac{1}{a}$$

  The residue at $z = a$ is equal to

$$P(z) = \frac{1}{z} \qquad P(a) = \frac{1}{a}$$

  Adding the results of the two residues, we get

$$h(-1) = 0$$

  In a similar manner, it can be shown that $h(n) = 0$ for $n < 0$

- For $n = 0$, C encloses only one pole, at $z = a$. Then

$$P(z) = 1$$

  evaluating the residue, we get

$$h(0) = 1$$

We conclude that the required unit-sample sequence is the following:

$$h(n) = \begin{cases} a^n & n \ge 0 \\ 0 & otherwise \end{cases}$$

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2 Computer Projects

1. Magnitude and phase response and singularity location. Find the program for the magnitude and phase response for given simple pole and zero and verify it on examples:

   (a) Real zero at $z = \frac{3}{4}$ an pole at $z = 0$. Repeat for zero at $z = \frac{4}{3}$. Explain the result.

   (b) Real pole at $z = -0.9$, zero at $z = 0$. Repeat by moving the zero from $z = 0$ to $z = -\frac{1}{0.9}$. Explain the result.

   (c) Complex-conjugate poles at $\rho = 0.95e^{\pm\frac{j\pi}{8}}$ and two zeros at $z = 0$.

   Solution: The following code is enough to compute and plot magnitude and phase response given the coefficients of the system function's numerator and denominator polynomials. At first we use `freqz` M-file applied to the vectors of coefficients `a` and `b` in order to compute the filter's transfer function. Next we plot the Pole/Zero pattern in a Z-plane. We need now to compute magnitude response and phase response, which is done by the program using the M-files `abs` and `angle`. The plot of these responses is done by using the `plot` M-file.

   ```
   % Computes the frequency response vector and returns the corresponding frequency vector
   % Default: 512 samples
   [h, w] = freqz(b, a);

   % Plots the poles and zeros on the Z-plane
   subplot(3, 1, 1);
   zplane(b, a);

   % Computes and plots magnitude response
   subplot(3, 1, 2);
   plot(w/pi, abs(h));
   xlabel('\omega/\pi'); ylabel('Magnitude')

   % Computes and plots phase response
   subplot(3, 1, 3);
   plot(w/pi, angle(h));
   xlabel('\omega/\pi'); ylabel('Phase, radians')
   ```

   (a) In this case we will assign the following values to the coefficient vectors prior using the M-file `freqz`:

   ```
   b = [1 -3/4];
   a = [1];
   ```

   The plots obtained for this pattern are shown in figure 8. Now we must execute the program for another pattern which is specified by assigning the following values to the vectors:

   ```
   b = [1 -4/3];
   a = [1];
   ```

   The plots for this new pattern are depicted in figure 9.

   (b) Now we will show another pattern in which there are poles out of the origin. We will begin assigning the following set of coefficients to the vectors:

   ```
   b = [1];
   a = [1 0.9];
   ```

   The plots corresponding to this new Pole/Zero pattern are illustrated in figure 10. Now we must move the zero from $z = 0$ to $z = -\frac{1}{0.9}$, this is done by adding a new coefficient to the vector associated to the numerator of the system function as follows:

   ```
   b = [1 -1/0.9];
   a = [1 0.9];
   ```

Plotting magnitude and phase responses for this patter using the program shown above generated the warning message "Warning: Requested axes limit range too small; rendering with minimum range allowed by machine precision." and the result was a plot of a flat magnitude response. We used an alternate program to verify this result, which is listed next:

```
w = 0:2*pi/512:2*pi;

% Computes unit sample response
[h, t] = impz(b, a);

% Computes transfer function
H = fft(h, 512);

% Plots the poles and zeros on the Z-plane
subplot(3, 1, 1);
zplane(b, a);

% Computes and plots magnitude response
subplot(3, 1, 2);
plot(w/pi, abs(H));
xlabel('\omega/\pi'); ylabel('Magnitude')

% Computes and plots phase response
subplot(3, 1, 3);
plot(w/pi, angle(H));
xlabel('\omega/\pi'); ylabel('Phase, radians')
```

The magnitude and phase responses obtaind in this way consider a range of frequencies from 0 to 2π and their plotes are shown in figure 11.

(c) Finally we consider a pattern which has two complex-conjugate poles inside the unit circle and the corresponding zeros at the origin. The vectors of coefficients in the original program are assigned in the following way:

```
b = [1];
a = [1 -1.9*cos(pi/8) 0.9025]
```

The plots of these singularities in the Z-plane and those for magnitude and phase response are shown in figure 12.
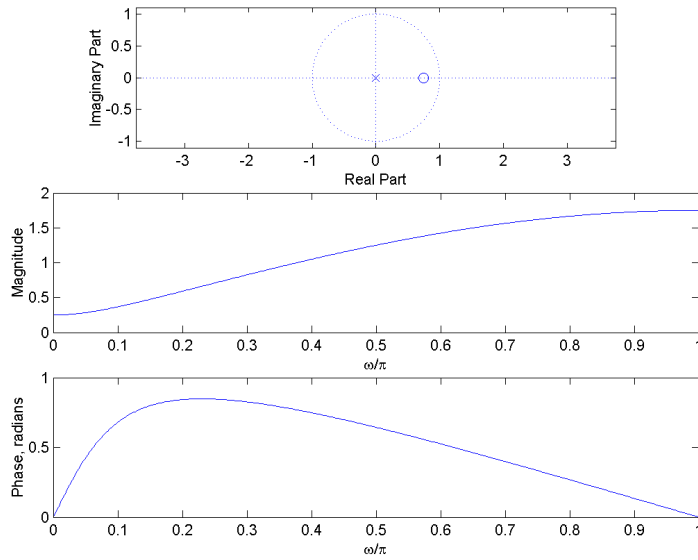
Figure 8: Magnitude and phase response for Pole/Zero pattern: zero at $z = \frac{3}{4}$, pole at $z = 0$.
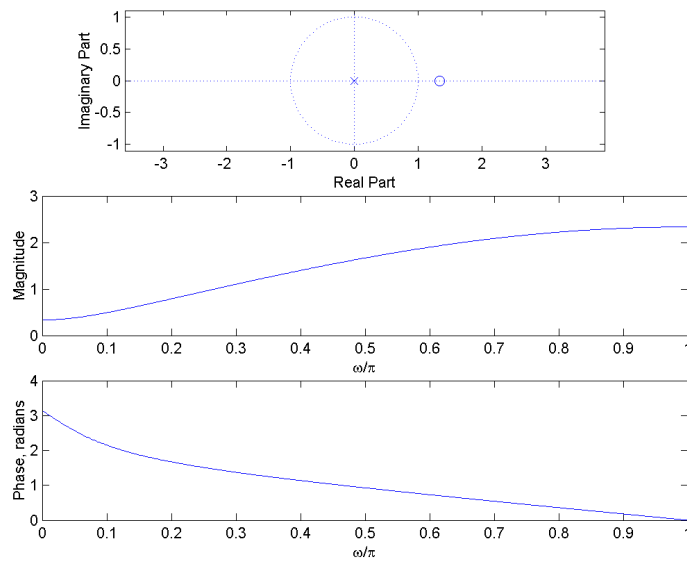

Figure 9: Magnitude and phase response for Pole/Zero pattern: zero at $z = \frac{4}{3}$, pole at $z = 0$.
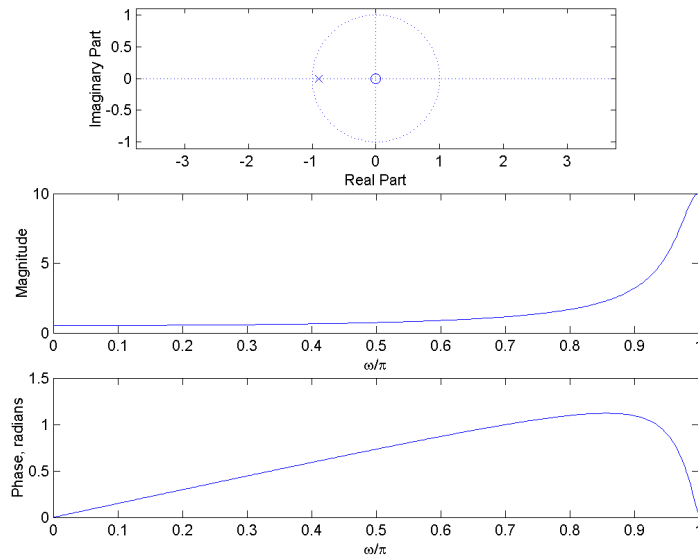
13

Figure 10: Magnitude and phase response for Pole/Zero pattern: zero at $z = 0$, pole at $z = 0.9$.
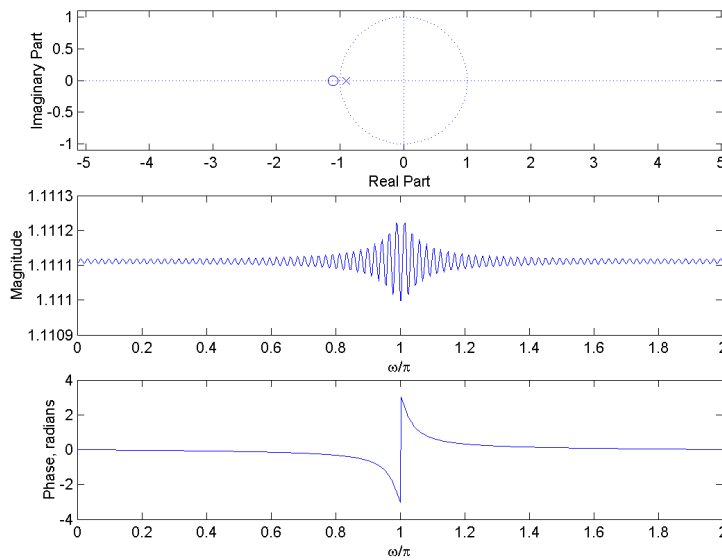

Figure 11: Magnitude and phase response for Pole/Zero pattern: zero at $z = -\frac{1}{0.9}$, pole at $z = 0.9$.
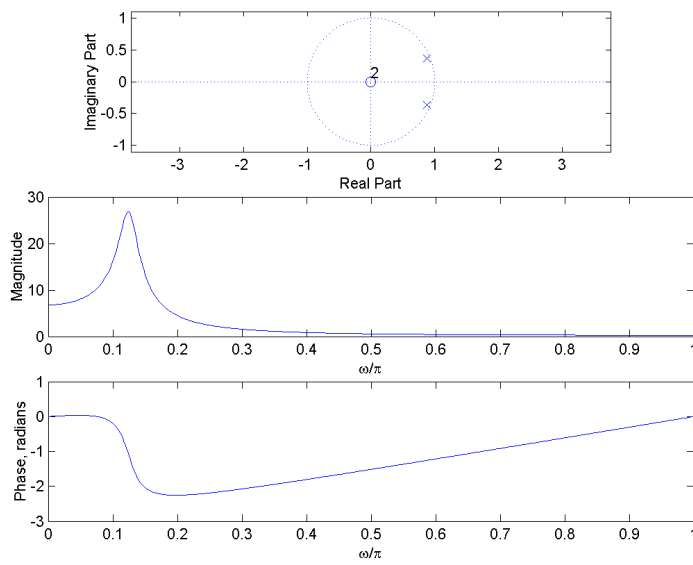
Figure 12: Magnitude and phase response for Pole/Zero pattern: zeros at $z = 0$, poles at $z = 0.95e^{\pm j\frac{\pi}{8}}$.