

Homework No. 3

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1 Problems

1. Evaluate and compare the 8-point DFT for the following sequences:

$$(a) x(n) = \begin{cases} 1 & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad (b) x(n) = \begin{cases} 1 & 0 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

(a) Since this is a finite duration, noncausal sequence, we will use the alternate sequence:

$$x(n) = \begin{cases} 1 & 0 \leq n \leq 3 \text{ or } 5 \leq n \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

therefore

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \quad 0 \leq k \leq N-1 \\ &= \sum_{n=0}^7 x(n)e^{-j\frac{2\pi}{8}nk} \quad 0 \leq k \leq 7 \\ &= x(0)e^{-j\frac{2\pi}{8}0k} + x(1)e^{-j\frac{2\pi}{8}1k} + x(2)e^{-j\frac{2\pi}{8}2k} + x(3)e^{-j\frac{2\pi}{8}3k} + x(4)e^{-j\frac{2\pi}{8}4k} + \\ &\quad + x(5)e^{-j\frac{2\pi}{8}5k} + x(6)e^{-j\frac{2\pi}{8}6k} + x(7)e^{-j\frac{2\pi}{8}7k} \\ &= e^{-j\frac{2\pi}{8}0k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}1k} + e^{-j\frac{2\pi}{8}3k} + e^{-j\frac{2\pi}{8}5k} + e^{-j\frac{2\pi}{8}7k} + e^{-j\frac{2\pi}{8}6k} \\ &= e^{-j\frac{2\pi}{8}k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + e^{-j\frac{2\pi}{8}2k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + e^{-j\frac{2\pi}{8}6k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + \\ &\quad + e^{-j\frac{2\pi}{8}6k} \\ &= \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] \left[e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}6k} \right] + e^{-j\frac{2\pi}{8}6k} \\ &= 2 \cos \left(\frac{2\pi}{8}k \right) \left[e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}6k} \right] + e^{-j\frac{2\pi}{8}6k} \quad 0 \leq k \leq 7 \end{aligned}$$

(b) In this case we have a causal sequence, so we will use the definition of the sequence directly

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \quad 0 \leq k \leq N-1$$

$$\begin{aligned}
&= \sum_{n=0}^7 x(n)e^{-j\frac{2\pi}{8}nk} \quad 0 \leq k \leq 7 \\
&= x(0)e^{-j\frac{2\pi}{8}0k} + x(1)e^{-j\frac{2\pi}{8}1k} + x(2)e^{-j\frac{2\pi}{8}2k} + x(3)e^{-j\frac{2\pi}{8}3k} + x(4)e^{-j\frac{2\pi}{8}4k} + \\
&\quad + x(5)e^{-j\frac{2\pi}{8}5k} + x(6)e^{-j\frac{2\pi}{8}6k} + x(7)e^{-j\frac{2\pi}{8}7k} \\
&= e^{-j\frac{2\pi}{8}0k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}1k} + e^{-j\frac{2\pi}{8}3k} + e^{-j\frac{2\pi}{8}4k} + e^{-j\frac{2\pi}{8}6k} + e^{-j\frac{2\pi}{8}5k} \\
&= e^{-j\frac{2\pi}{8}k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + e^{-j\frac{2\pi}{8}2k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + e^{-j\frac{2\pi}{8}5k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + \\
&\quad + e^{-j\frac{2\pi}{8}5k} \\
&= \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] \left[e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}5k} \right] + e^{-j\frac{2\pi}{8}5k} \\
&= 2 \cos\left(\frac{2\pi}{8}k\right) \left[e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}5k} \right] + e^{-j\frac{2\pi}{8}5k} \quad 0 \leq k \leq 7
\end{aligned}$$

As we can see these results are so similar, they differ only in the terms $e^{-j\frac{2\pi}{8}5k}$ and $e^{-j\frac{2\pi}{8}6k}$. This situation makes sense due to the sequence in (b) is, actually, the sequence in (a) but delayed 3 samples.

2. Determine and sketch the discrete-time sequence to be processed to find 8-point DFT when the following sequences are given:

$$(a) x(n) = \begin{cases} 1 & n = -1 \\ 0 & \text{otherwise} \end{cases} \quad (b) x(n) = \begin{cases} 3 - |n| & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$(c) x(n) = \begin{cases} n & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

- (a) The computation of $\tilde{x}(n)$ is as follows:

$$\begin{aligned}
\tilde{x}(n) &= \sum_{m=-\infty}^{\infty} x(n+mN) = \sum_{m=-\infty}^{\infty} x(n+m8) \\
\tilde{x}(0) &= \sum_{m=-\infty}^{\infty} x(0+m8) = \dots + x(0-1 \cdot 8) + x(0) + x(0+1 \cdot 8) + \dots \\
&= \dots + x(-8) + x(0) + x(8) + \dots = 0 \\
\tilde{x}(1) &= \sum_{m=-\infty}^{\infty} x(1+m8) = \dots + x(1-1 \cdot 8) + x(1) + x(1+1 \cdot 8) + \dots \\
&= \dots + x(-7) + x(1) + x(9) + \dots = 0 \\
\tilde{x}(2) &= \sum_{m=-\infty}^{\infty} x(2+m8) = \dots + x(2-1 \cdot 8) + x(2) + x(2+1 \cdot 8) + \dots \\
&= \dots + x(-6) + x(2) + x(10) + \dots = 0 \\
\tilde{x}(3) &= \sum_{m=-\infty}^{\infty} x(3+m8) = \dots + x(3-1 \cdot 8) + x(3) + x(3+1 \cdot 8) + \dots \\
&= \dots + x(-5) + x(3) + x(11) + \dots = 0 \\
\tilde{x}(4) &= \sum_{m=-\infty}^{\infty} x(4+m8) = \dots + x(4-1 \cdot 8) + x(4) + x(4+1 \cdot 8) + \dots
\end{aligned}$$

$$\begin{aligned}
&= \dots + x(-4) + x(4) + x(12) + \dots = 0 \\
\tilde{x}(5) &= \sum_{m=-\infty}^{\infty} x(5 + m8) = \dots + x(5 - 1 \cdot 8) + x(5) + x(5 + 1 \cdot 8) + \dots \\
&= \dots + x(-3) + x(5) + x(13) + \dots = 0 \\
\tilde{x}(6) &= \sum_{m=-\infty}^{\infty} x(6 + m8) = \dots + x(6 - 1 \cdot 8) + x(6) + x(6 + 1 \cdot 8) + \dots \\
&= \dots + x(-2) + x(6) + x(14) + \dots = 0 \\
\tilde{x}(7) &= \sum_{m=-\infty}^{\infty} x(7 + m8) = \dots + x(7 - 1 \cdot 8) + x(7) + x(7 + 1 \cdot 8) + \dots \\
&= \dots + x(-1) + x(7) + x(15) + \dots = 1
\end{aligned}$$

Therefore the required discrete time sequence is the following:

$$x(n) = \begin{cases} 1 & n = 8m - 1 \quad \forall m \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

(b) In this case the computation of $\tilde{x}(n)$ is as follows:

$$\begin{aligned}
\tilde{x}(n) &= \sum_{m=-\infty}^{\infty} x(n + mN) = \sum_{m=-\infty}^{\infty} x(n + m8) \\
\tilde{x}(0) &= \sum_{m=-\infty}^{\infty} x(0 + m8) = \dots + x(0 - 1 \cdot 8) + x(0) + x(0 + 1 \cdot 8) + \dots \\
&= \dots + x(-8) + x(0) + x(8) + \dots = 3 \\
\tilde{x}(1) &= \sum_{m=-\infty}^{\infty} x(1 + m8) = \dots + x(1 - 1 \cdot 8) + x(1) + x(1 + 1 \cdot 8) + \dots \\
&= \dots + x(-7) + x(1) + x(9) + \dots = 2 \\
\tilde{x}(2) &= \sum_{m=-\infty}^{\infty} x(2 + m8) = \dots + x(2 - 1 \cdot 8) + x(2) + x(2 + 1 \cdot 8) + \dots \\
&= \dots + x(-6) + x(2) + x(10) + \dots = 1 \\
\tilde{x}(3) &= \sum_{m=-\infty}^{\infty} x(3 + m8) = \dots + x(3 - 1 \cdot 8) + x(3) + x(3 + 1 \cdot 8) + \dots \\
&= \dots + x(-5) + x(3) + x(11) + \dots = 0 \\
\tilde{x}(4) &= \sum_{m=-\infty}^{\infty} x(4 + m8) = \dots + x(4 - 1 \cdot 8) + x(4) + x(4 + 1 \cdot 8) + \dots \\
&= \dots + x(-4) + x(4) + x(12) + \dots = 0 \\
\tilde{x}(5) &= \sum_{m=-\infty}^{\infty} x(5 + m8) = \dots + x(5 - 1 \cdot 8) + x(5) + x(5 + 1 \cdot 8) + \dots \\
&= \dots + x(-3) + x(5) + x(13) + \dots = 0 \\
\tilde{x}(6) &= \sum_{m=-\infty}^{\infty} x(6 + m8) = \dots + x(6 - 1 \cdot 8) + x(6) + x(6 + 1 \cdot 8) + \dots
\end{aligned}$$

$$\begin{aligned}
&= \dots + x(-2) + x(6) + x(14) + \dots = 1 \\
\tilde{x}(7) &= \sum_{m=-\infty}^{\infty} x(7 + m8) = \dots + x(7 - 1 \cdot 8) + x(7) + x(7 + 1 \cdot 8) + \dots \\
&= \dots + x(-1) + x(7) + x(15) + \dots = 2
\end{aligned}$$

Therefore, the sequence we are looking for is:

$$x(n) = \begin{cases} 3 & n = 8m \\ 2 & n = 8m + 1 \text{ or } n = 8m + 7 \\ 1 & n = 8m + 2 \text{ or } n = 8m + 6 \\ 0 & \text{otherwise} \end{cases} \quad \forall m \in \mathbb{Z}$$

(c) In this case the computation of $\tilde{x}(n)$ is as follows:

$$\begin{aligned}
\tilde{x}(n) &= \sum_{m=-\infty}^{\infty} x(n + mN) = \sum_{m=-\infty}^{\infty} x(n + m8) \\
\tilde{x}(0) &= \sum_{m=-\infty}^{\infty} x(0 + m8) = \dots + x(0 - 1 \cdot 8) + x(0) + x(0 + 1 \cdot 8) + \dots \\
&= \dots + x(-8) + x(0) + x(8) + \dots = 0 \\
\tilde{x}(1) &= \sum_{m=-\infty}^{\infty} x(1 + m8) = \dots + x(1 - 1 \cdot 8) + x(1) + x(1 + 1 \cdot 8) + \dots \\
&= \dots + x(-7) + x(1) + x(9) + \dots = 1 \\
\tilde{x}(2) &= \sum_{m=-\infty}^{\infty} x(2 + m8) = \dots + x(2 - 1 \cdot 8) + x(2) + x(2 + 1 \cdot 8) + \dots \\
&= \dots + x(-6) + x(2) + x(10) + \dots = 2 \\
\tilde{x}(3) &= \sum_{m=-\infty}^{\infty} x(3 + m8) = \dots + x(3 - 1 \cdot 8) + x(3) + x(3 + 1 \cdot 8) + \dots \\
&= \dots + x(-5) + x(3) + x(11) + \dots = 3 \\
\tilde{x}(4) &= \sum_{m=-\infty}^{\infty} x(4 + m8) = \dots + x(4 - 1 \cdot 8) + x(4) + x(4 + 1 \cdot 8) + \dots \\
&= \dots + x(-4) + x(4) + x(12) + \dots = 0 \\
\tilde{x}(5) &= \sum_{m=-\infty}^{\infty} x(5 + m8) = \dots + x(5 - 1 \cdot 8) + x(5) + x(5 + 1 \cdot 8) + \dots \\
&= \dots + x(-3) + x(5) + x(13) + \dots = -3 \\
\tilde{x}(6) &= \sum_{m=-\infty}^{\infty} x(6 + m8) = \dots + x(6 - 1 \cdot 8) + x(6) + x(6 + 1 \cdot 8) + \dots \\
&= \dots + x(-2) + x(6) + x(14) + \dots = -2 \\
\tilde{x}(7) &= \sum_{m=-\infty}^{\infty} x(7 + m8) = \dots + x(7 - 1 \cdot 8) + x(7) + x(7 + 1 \cdot 8) + \dots \\
&= \dots + x(-1) + x(7) + x(15) + \dots = -1
\end{aligned}$$

Therefore, the sequence we are looking for is:

$$x(n) = \begin{cases} 3 & n = 8m + 3 \\ 2 & n = 8m + 2 \\ 1 & n = 8m + 1 \\ -3 & n = 8m + 5 \\ -2 & n = 8m + 6 \\ -1 & n = 8m + 7 \\ 0 & \text{otherwise} \end{cases} \quad \forall m \in \mathbb{Z}$$

3. Compute N-point DFT of each of the following sequences:

$$(a) x(n) = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (b) x(n) = \begin{cases} 1 & n = 0 \\ -1 & n = M \end{cases} \quad N = 8, M = 5$$

Solution:

This time we have two causal sequences, so we will make direct use of the values of the sequence as coefficients for the N-point DFT as follows:

(a)

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \quad 0 \leq k \leq N-1 \\ &= \sum_{n=0}^7 x(n)e^{-j\frac{2\pi}{8}nk} \quad 0 \leq k \leq 7 \\ &= x(0)e^{-j\frac{2\pi}{8}0k} + x(1)e^{-j\frac{2\pi}{8}1k} + x(2)e^{-j\frac{2\pi}{8}2k} + x(3)e^{-j\frac{2\pi}{8}3k} + x(4)e^{-j\frac{2\pi}{8}4k} + \\ &\quad + x(5)e^{-j\frac{2\pi}{8}5k} + x(6)e^{-j\frac{2\pi}{8}6k} + x(7)e^{-j\frac{2\pi}{8}7k} \\ &= 1 + e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}3k} + e^{-j\frac{2\pi}{8}4k} \\ &= 1 + e^{-j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}3k} + e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}4k} \\ &= 1 + e^{-j\frac{2\pi}{8}2k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] + e^{-j\frac{2\pi}{8}3k} \left[e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k} \right] \\ &= 1 + e^{-j\frac{2\pi}{8}2k} 2 \cos\left(\frac{2\pi}{8}k\right) + e^{-j\frac{2\pi}{8}3k} 2 \cos\left(\frac{2\pi}{8}k\right) \\ &= 1 + 2 \cos\left(\frac{2\pi}{8}k\right) \left[e^{-j\frac{2\pi}{8}2k} + e^{-j\frac{2\pi}{8}3k} \right] \quad 0 \leq k \leq 7 \end{aligned}$$

(b)

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \quad 0 \leq k \leq N-1 \\ &= \sum_{n=0}^7 x(n)e^{-j\frac{2\pi}{8}nk} \quad 0 \leq k \leq 7 \\ &= x(0)e^{-j\frac{2\pi}{8}0k} + x(1)e^{-j\frac{2\pi}{8}1k} + x(2)e^{-j\frac{2\pi}{8}2k} + x(3)e^{-j\frac{2\pi}{8}3k} + x(4)e^{-j\frac{2\pi}{8}4k} + \\ &\quad + x(5)e^{-j\frac{2\pi}{8}5k} + x(6)e^{-j\frac{2\pi}{8}6k} + x(7)e^{-j\frac{2\pi}{8}7k} \\ &= x(0)e^{-j\frac{2\pi}{8}0k} + x(5)e^{-j\frac{2\pi}{8}5k} \\ &= 1 - e^{-j\frac{2\pi}{8}5k} \\ &= 1 - \left[\cos\left(-\frac{2\pi}{8}5k\right) + j \sin\left(-\frac{2\pi}{8}5k\right) \right] \\ &= 1 - \left[\cos\left(\frac{2\pi}{8}5k\right) - j \sin\left(\frac{2\pi}{8}5k\right) \right] \end{aligned}$$

$$= \left[1 - \cos\left(\frac{2\pi}{8}5k\right) \right] + j \sin\left(-\frac{2\pi}{8}5k\right) \quad 0 \leq n \leq 7$$

4. Perform the circular convolution of the following two sequences:

$$x(n) = \begin{cases} 2 & n=0 \\ 1 & n=1 \\ 2 & n=2 \\ 1 & n=3 \end{cases} \quad h(n) = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 4 & n=3 \end{cases}$$

Solution: If we carried out linear convolution of both sequences, we would get a sequence whose length would be seven samples. We will pad both sequences with zeros in order that circular convolution to be equal to linear convolution. These modified sequences are the following:

$$x(n) = \begin{cases} 2 & n=0 \\ 1 & n=1 \\ 2 & n=2 \\ 1 & n=3 \\ 0 & 4 \leq n \leq 6 \end{cases} \quad h(n) = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 4 & n=3 \\ 0 & 4 \leq n \leq 6 \end{cases}$$

Next we compute $y(n) = x(n) \circledast h(n)$.

$$\begin{aligned} y(0) &= x(0)h(0) + x(1)h(6) + x(2)h(5) + x(3)h(4) + x(4)h(3) + x(5)h(2) + x(6)h(1) \\ &= 2 + 0 + 0 + 0 + 0 + 0 + 0 = 2 \\ y(1) &= x(0)h(1) + x(1)h(0) + x(2)h(6) + x(3)h(5) + x(4)h(4) + x(5)h(3) + x(6)h(2) \\ &= 4 + 1 + 0 + 0 + 0 + 0 + 0 = 5 \\ y(2) &= x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(6) + x(4)h(5) + x(5)h(4) + x(6)h(3) \\ &= 6 + 2 + 2 + 0 + 0 + 0 + 0 = 10 \\ y(3) &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) + x(4)h(6) + x(5)h(5) + x(6)h(4) \\ &= 8 + 3 + 4 + 1 + 0 + 0 + 0 = 16 \\ y(4) &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) + x(5)h(6) + x(6)h(5) \\ &= 0 + 4 + 6 + 2 + 0 + 0 + 0 = 12 \\ y(5) &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) + x(6)h(6) \\ &= 0 + 0 + 8 + 3 + 0 + 0 + 0 = 11 \\ y(6) &= x(0)h(6) + x(1)h(5) + x(2)h(4) + x(3)h(3) + x(4)h(2) + x(5)h(1) + x(6)h(0) \\ &= 0 + 0 + 0 + 4 + 0 + 0 + 0 = 4 \end{aligned}$$

2 Computer Projects

1. Padding with zeros. Interpolating the DFT by padding the time domain sequence $\{x(n)\}$ with zeros:

$$x(n) = r^n \cos(\omega_0 n), \quad \text{for } 0 \leq n \leq 63$$

$$r = 0.9, \quad \omega_0 = \frac{9\pi}{64}$$

(a) Compute the 64-point DFT and plot the 33 points of magnitude and phase response corresponding to

$$0 \leq \omega \leq \pi$$

- (b) Generate the 128-point sequence $\{y(n)\}$ by padding $\{x(n)\}$ with zeros:

$$y(n) = \begin{cases} x(n) & 0 \leq n \leq 63 \\ 0 & 64 \leq n \leq 127 \end{cases}$$

Compute the 128-point DFT and plot the 65 points of magnitude and phase response corresponding to

$$0 \leq \omega \leq \pi$$

Compare the results in both cases

Solution:

- (a) The MATLAB program which computes the 64-point DFT is shown next. The first section of the program is the following:

```
% Parameters for the sequence x(n)
r = 0.9;
w0 = 9*pi/4;

% Number of points in the x(n) sequence
N = 63;
% Number of points to be used to compute DFT
M = 64;

% Generates the sequence 0, 1, 2, ..., N-1
n = 0:1:N-1;

% Generates the sequence x(n) whose DFT we want to compute
x = r.^n .* cos(w0 * n);
```

At first this code assigns the parameters ω_0 and r which are needed to compute the sequence whose DFT we want to compute. The next assignments are the length of the sequence $x(n)$ and the number of points used to compute DFT. The last sentence in this section calculates each of the values of the sequence. The last section of the code is the one which performs the real work:

```
% Computes M point DFT
U = fft(x, M);

% Plots the time domain sequence
subplot(3, 1, 1);
stem(n, x);
title('x(n)');

% Plots magnitude response over 0 < w < pi
T = abs(U);

subplot(3, 1, 2);
stem(0:32, T(1:33))
title('Magnitude of the DFT samples')
xlabel('Frequency index k');
ylabel('Magnitude');

% Plots phase response over 0 < w < pi
P = angle(U);

subplot(3, 1, 3);
```

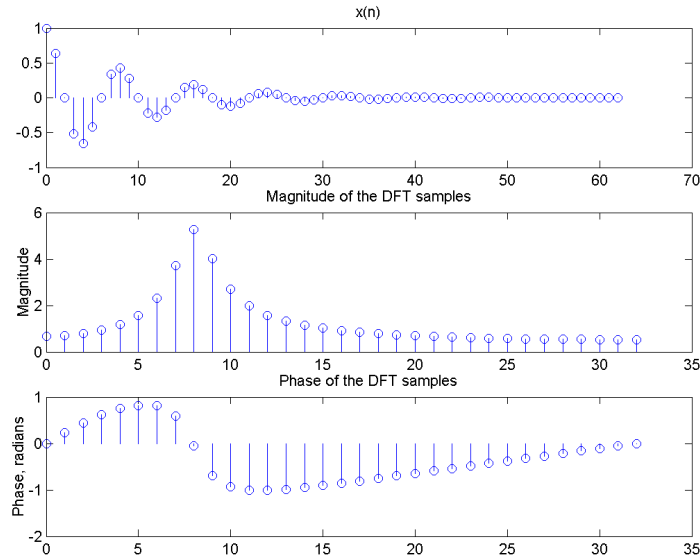


Figure 1: Magnitude response and phase response for $x(n) = 0.9^n \cos(\frac{9\pi}{64}n)$ using a 64-point DFT.

```
stem(0:32, P(1:33))
title('Phase of the DFT samples')
%xlabel('Frequency index k');
ylabel('Phase, radians');
```

The `fft` m-file carries out the job of computing DFT given a number of points. The `abs` and `angle` m-files use the result of the DFT procedure and compute, respectively, magnitude response and phase response. Figure 1 depicts the plots for these responses in the interval $[0, \pi]$, which in this case corresponds to 33 indexes.

- (b) For the case of a 128-point DFT the code is the same, but the only difference is the sentence which assigns the number of points to use in DFT to the variable `M`. Figure 2 illustrates the plots of magnitude and phase response.

Notice that in this case the plots are more accurate since we have considered a greater number of points. In general this will cause that the sequence obtained by IDFT from DFT has a greater period and would reduce aliasing a bit, in case of infinite duration sequence, or the total disappearance of it in case of a finite duration sequence.

2. Convolution

- (a) Perform the linear convolution with DFT.
 (b) Perform the circular convolution with DFT and demonstrate the aliasing phenomenon produced by choosing `N` too small. Let:

$$x(n) = \begin{cases} 1 & 0 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases} \quad h(n) = \begin{cases} 1 & 0 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

- (a) Computing linear convolution of two sequences $x(n)$ and $h(n)$ using DFT is not difficult. At first we must realize that the length of these sequences might be different (say m and n ,

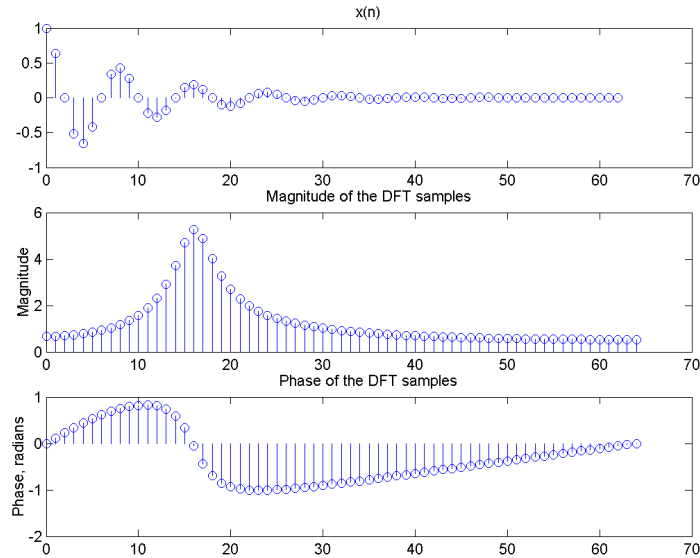


Figure 2: Magnitude response and phase response for $x(n) = 0.9^n \cos(\frac{9\pi}{64}n)$ using a 128-point DFT.

respectively) and in this case the linear convolution would have a length $l = m + n - 1$. Next we compute the l -point DFT for each sequence padding zeros as necessary, form the product of these DFTs and apply IDFT to this product. Since we have used an appropriate value for the number of points we will get a result that matches with the direct application of the convolution operator. The following MATLAB code applies this procedure to compute linear convolution and plots the sequence resulting from it along with the result of application of the convolution operator directly:

```
% Linear Convolution via the DFT

% Read in the two sequences
x = input('Type in x(n) sequence = ');
h = input('Type in h(n) sequence = ');

% Determine the length of the result of the convolution
L = length(x) + length(h) - 1;

% Compute the DFTs by zero-padding
Xe = fft(x, L);
He = fft(h, L);

% Determine IDFT of the product
y1 = ifft(Xe.*He);

% Determine direct linear convolution of the input
% sequences
y2 = conv(x, h);

% Plot the sequence generated by DFT-based
% convolution
k = 0:1:L-1
```

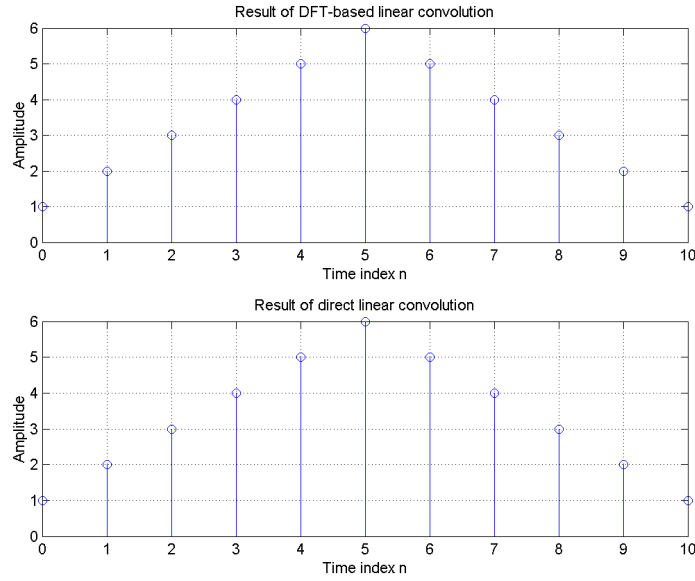


Figure 3: Comparison of linear convolution of two sequences by using DFT and the operator.

```
subplot(2, 1, 1)
stem(k, y1); grid
xlabel('Time index n'); ylabel('Amplitude');
title('Result of DFT-based linear convolution');
```

```
% Plot the sequence generated by direct convolution
subplot(2, 1, 2)
stem(k, y2); grid
xlabel('Time index n'); ylabel('Amplitude');
title('Result of direct linear convolution');
```

The plots produced by this program corresponding to the sequences $x(n)$ and $h(n)$ specified above are depicted in figure 3. Notice that the results are exactly the same.

- (b) In this case we are interested in computing circular convolution of the two sequences $x(n)$ and $h(n)$. We can do this by using DFT sampled at certain number of points, say n . For instance, our two sequences have a length of 6 samples so we can use several values for n in DFT starting with 6 and analyze the results. To obtain circular convolution we must compute n -point DFT for each sequence, multiply these results and apply n -point IDFT to this product. The following MATLAB code will carry out this procedure:

```
% Circular Convolution via the DFT

% Read in the two sequences
x = input('Type in x(n) sequence = ');
h = input('Type in h(n) sequence = ');

N = length(x);

% Compute N-point DFTs
Xe = fft(x, N);
```

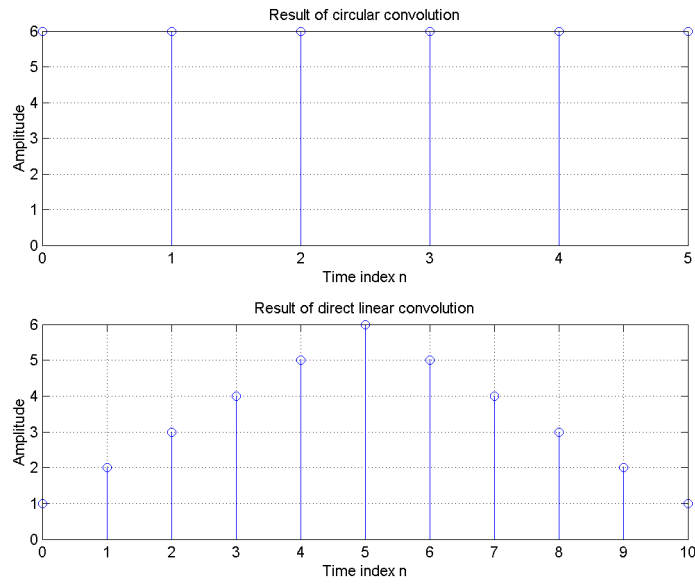


Figure 4: Circular convolution of $x(n)$ and $h(n)$ ($n = 6$).

```

He = fft(h, N);

% Determine IDFT of the product
y1 = ifft(Xe.*He, N);

% Determine direct linear convolution of the input
% sequences
y2 = conv(x, h);

subplot(2, 1, 1)
stem(0:1:length(y1) - 1, y1); grid
xlabel('Time index n'); ylabel('Amplitude');
title('Result of circular convolution');

% Plot the sequence generated by direct convolution
subplot(2, 1, 2)
stem(0:1:length(y2) - 1, y2); grid
xlabel('Time index n'); ylabel('Amplitude');
title('Result of direct linear convolution');

```

Changing the value of the variable N allows us to get different results. If N is small enough the effects of aliasing will be more notorious, as N increases the effects of aliasing decrease and they will eventually disappear. The former program was applied to the sequences specified above, changing the value of N each time. The results are shown in figures 4–6.

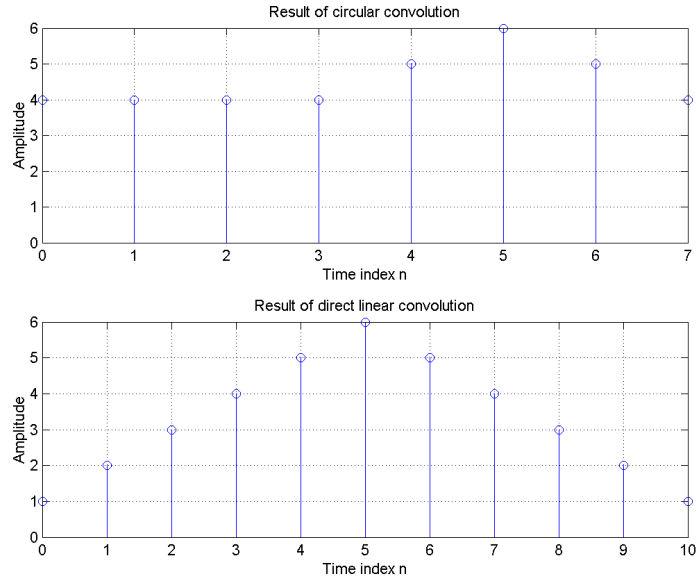


Figure 5: Circular convolution of $x(n)$ and $h(n)$ ($n = 8$).

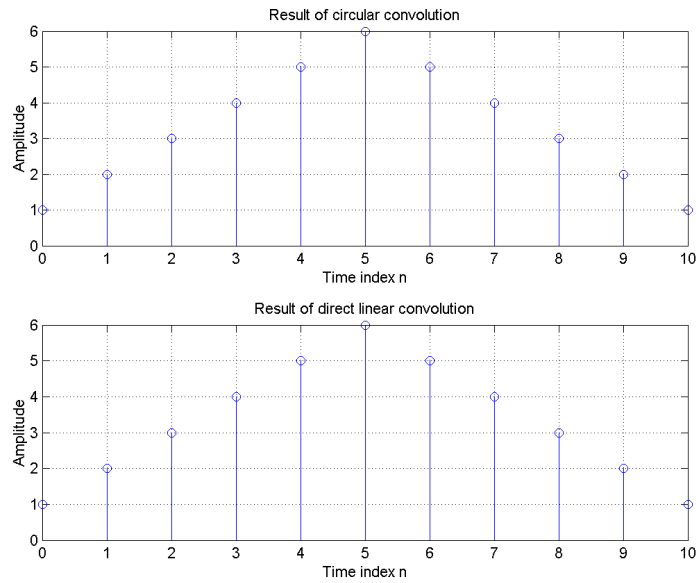


Figure 6: Circular convolution of $x(n)$ and $h(n)$ ($n = 11$).